“The Pre-History of the Two Black Hole Collision Problem”

Invited Talk

History of Numerical Relativity Session

American Physical Society

Columbus, OH

April 15, 2018

Dr. Larry Smarr

Director, California Institute for Telecommunications and Information Technology

Harry E. Gruber Professor,

Dept. of Computer Science and Engineering

Jacobs School of Engineering, UCSD

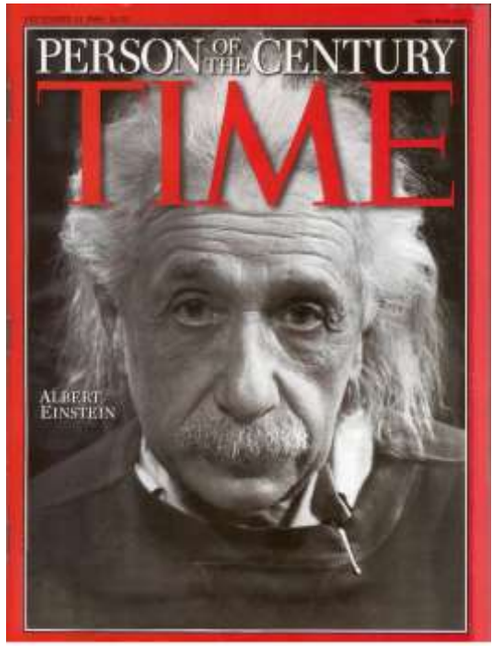
http://lsmarr.calit2.net

Abstract

From the first mathematical solution of Einstein's equations of General Relativity, representing what we now know as a black hole, in 1918 to the observation of the gravitational radiation from two colliding black holes in 2015 was almost 100 years. I will give a brief history of the mathematical and computational developments, up to the 1970s when the first computational solution of Einsteins equations for two black holes colliding head-on was obtained. The 1920s saw the equation of motion posed, the 1930s envisioned the two-body problem, the 1940s set up the Cauchy problem, the 1950s conceived of numerical relativity, the 1960s witnessed the first numerical solutions, and the 1970s produced the first numerical collision with generation of gravitational radiation.

The Two Black Hole Collision Is a One Hundred Year Physics Research Problem

1915
Einstein Field Equations
Schwarzschild Solution for One Black Hole



2015
Gravitational Radiation From Two Colliding Black Holes Detected



This Talk Will Cover the First 60 Years

The Spherical Solution of Einstein's Field Equations, the Schwarzschild Black Hole, Was Derived in 1915



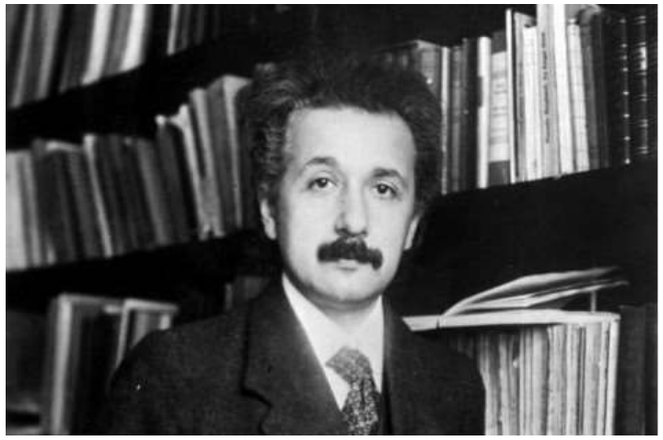
"On the Gravitational Field of a Point Mass in Einstein's Theory,"
Proceedings of the Prussian Academy of Sciences, 424 (1916)

Über das Gravitationsfeld eines Massenpunktes nach der EINSTEINschen Theorie.

Von K. SCHWARZSCHILD.

(Vorgelegt am 13. Januar 1916 [s. oben S. 42].)

Five Years Later
My Father Was Born



I have read your paper with the utmost interest.
I had not expected that one could formulate the exact solution
of the problem in such a simple way.
I liked very much your mathematical treatment of the subject.
Next Thursday I shall present the work to the Academy
with a few words of explanation.
—Albert Einstein letter to Karl Schwarzschild (1916)

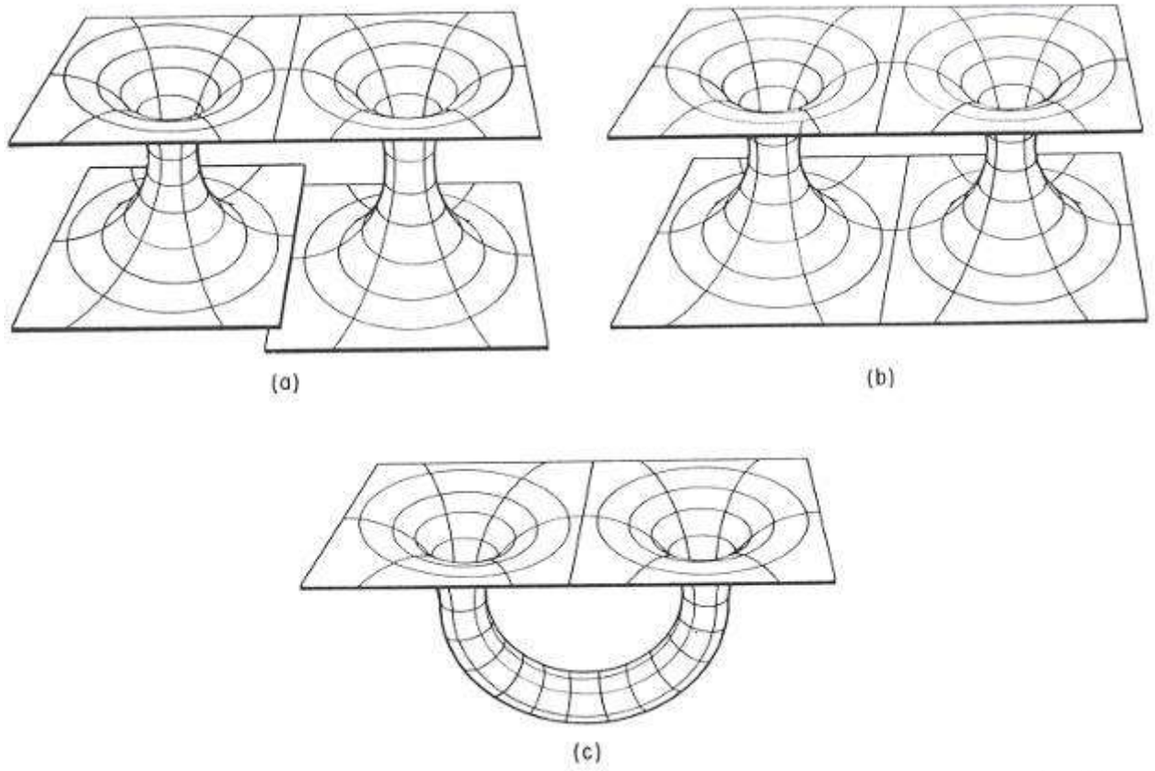
Einstein and Rosen Pose Non-Singular Two Body Problem in 1935

JULY 1, 1935 PHYSICAL REVIEW VOLUME 48

The Particle Problem in the General Theory of Relativity

A. EINSTEIN AND N. ROSEN, *Institute for Advanced Study, Princeton*

Hahn and Lindquist, Ann.Phys., 29, p. 307 (1964)



André Lichnerowicz in 1944
Sets Up 2 Body Problem and Foresees Numerical Relativity

- **Sets up Cauchy Problem in 3+1 Form ($\partial_t K^i_j$=…)**
- **Studies Minimal Surfaces and Finds:**
  - **K=0 Means Minimal if Shift Vector is Zero**
  - **Elliptic Lapse Equation**
  - **Normal Congruence Behaves Like Irrotational Incompressible Fluid**
- **Finds Elliptic Eqn. for 3-Metric Conformal Factor**
- **Sets Up n-Body Problem with Matter:**
  - **Time Symmetric Initial Data for Conformally Flat 3-Space**
  - **Geodesic Normal Gauge for Evolution**
  - **Uses Matter Instead of non-Euclidean Topology as Body Models**
  - **Solves for Conformal Factor and Exhibits Interaction Energy**
- **"A de telles donnés correspondra une solution *rigoureuse* de ce problème, dont l'évolution dans le temps sera régie par les équations et pourra être obtenue par une intégration numérique de ces équations."**

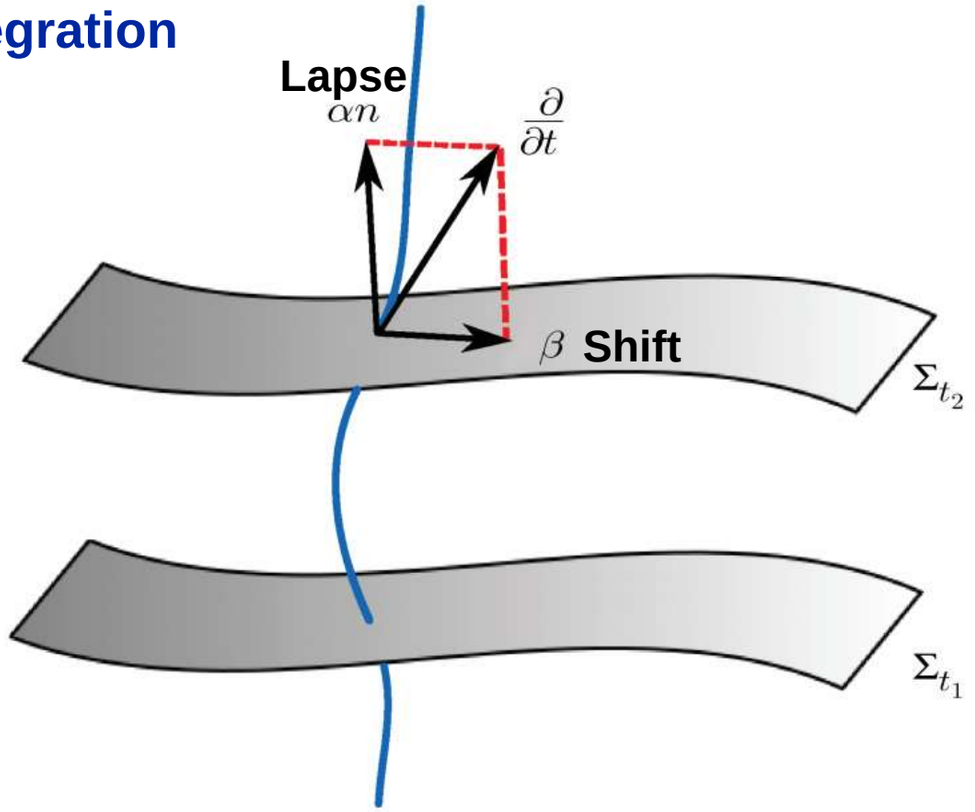
Five Years Later



"L'intégration des Équations de la Gravitation Relativiste et le Problème des n Corps"
Journal de mathematiques pures et appliques 23, 37 (1944)

The Cauchy Evolution of Initial Data

- 1944 Lichnerowicz
  - 3+1 Decomposition, Idea of Numerical Integration
- 1956 Choquet-Bruhat
  - Formalizes Cauchy Problem
- 1957 DeWitt, Misner
  - Concept of Numerical Relativity
- 1959 Wheeler, Misner
  - Geometrodynamics and Superspace
- 1961 Arnowitt, Deser, & Misner
  - Canonical Decomposition



Source: Holst, et al. Bull. AMS (2016)

Chapel Hill Conference on the Role of Gravitation in Physics 1957

- **Bryce DeWitt asked if the Cauchy problem is now understood sufficiently to be put on an electronic computer for actual calculation.**

- **Charles Misner answered that he had computed initial data for two Einstein-Rosen throats that "can be interpreted as two particles which are non-singular… These partial differential equations, although very difficult, can then in principle be put on a computer."**

- **Misner thinks that one can now give initial conditions so that one would expect to get gravitational radiation, and computers could be used for this.**

APS physics

Session Y06: The Chapel Hill Conference and its Role in the Renaissance of General Relativity
1:30 PM–3:18 PM, Tuesday, April 17, 2018
Room: B130

APRIL MEETING 2018

Conference on the Role of Gravitation in Physics,
Wright Air Development Center Technical Report 57-216 (1957)
http://www.edition-open-sources.org/media/sources/5/Sources5.pdf

The First Crisp Definition of Numerical Relativity

- **Misner Summarizes—**
  - **”First we assume that have a computing machine better than anything we have now, and many programmers and a lot of money, and you want to look at a nice pretty solution of the Einstein equations. The computer wants to know from you what are the values of $g_{\mu\nu}$ and $\partial_t g_{\mu\nu}$ at some initial surface. Mme. Foures has told us that to get these initial conditions you must specify something else and hand over that problem, the problem of the initial values, to a smaller computer first, before you start on what Lichnerowicz called the evolutionary problem. The small computer would prepare the initial conditions for the big one. Then the theory, while not guaranteeing solutions for the whole future, says that it will be some finite time before anything blows up.”**

Note Supercomputers Are Still Using Vacuum Tubes at This Time!

Conference on the Role of Gravitation in Physics,
Wright Air Development Center Technical Report 57-216 (1957)
http://www.edition-open-sources.org/media/sources/5/Sources5.pdf

Bryce DeWitt Foresees the Three Major Conceptual Challenges of Numerical Relativity

- **“Bryce DeWitt pointed out some difficulties encountered in high-speed computational techniques. Problems would arise in applying computers to gravitational radiation, since you don’t want the radiation to move quickly out of the range of your computer.” --page 83 of 1957 Chapel Hill Conference**

- **“Bryce saw clearly in 1957…the conceptual problems in simultaneously worrying about”:**
  - **The Computer Algorithm**
  - **The Structure of Space-Time**
  - **The Coordinate System**

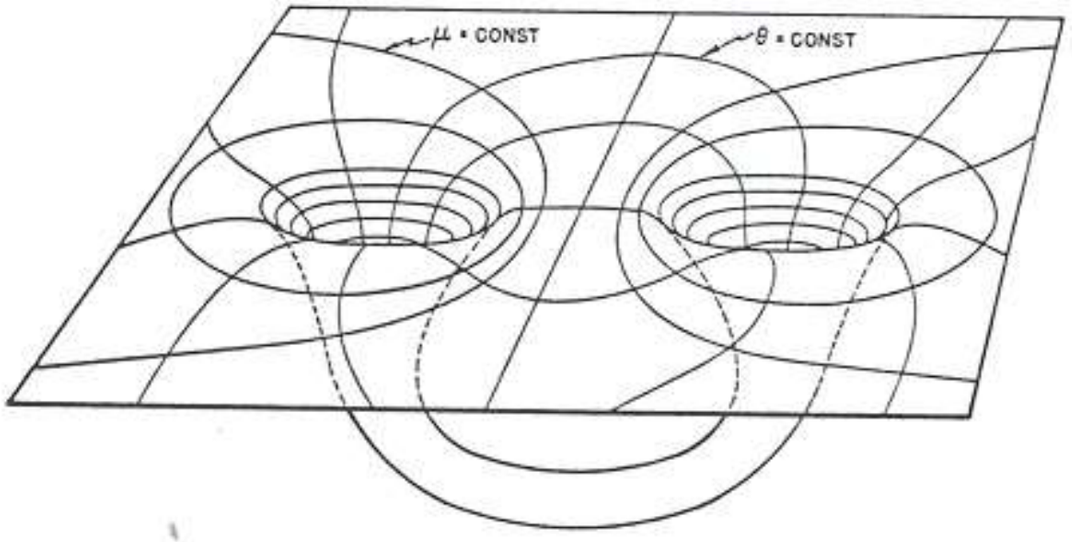
Source: Larry Smarr, *The Contribution Of Bryce DeWitt To Classical General Relativity* In Ahead of His Time: Bryce S. DeWitt. 1984, ed. S. Christensen

Geometrodynamics of Wormholes
"Mass Without Mass"

"Geometrodynamics and the Problem of Motion"

"The evolution in time of the wormhole 3-geometry thus specified can be found in the beginning by power series expansion and thereafter by electronic computation. The intrinsic geometry of the resulting 4-space is completely determinate, regardless of the freedom of choice that is open as to the coordinate system to be used to describe that geometry. This geometry contains within itself the story as the change of the distance L between the throats with time and the generation of gravitational waves by the two equal masses as they are accelerated towards each other."

--John Archibald Wheeler, Rev. Mod. Phys. 33, 70 (1961)



Misner, Phys. Rev., 118, p. 1110 (1960)

Hahn and Lindquist 1964
"The Two Body Problem in Geometrodynamics"

- Conceptually Studying Causality and Area of Throats
- Black Hole is not a Term until Four Years Later
- Used Misner Coordinates
  - Good Near Throats
  - Terrible at Large Distances
  - Mesh Size 51x151
- Used Geodesic Normal Coordinates
- Initial Data Represented "Already Merged" Black Holes (μ_o=1.6)
- Used IBM 7090 (~0.3 MFLOPS or ~1 Millionth the Speed of an iPhone 7)
  - Integrated Very Short Time to Future (<0.3M)
- Proof of Principle that Numerical Relativity Worked

Hahn and Lindquist, Ann.Phys., 29, p. 304 (1964)

Why Did I Attack the Two Black Hole Problem in 1972?

- **Bryce Said “Just Do It!”**
- **Explore Geometrodynamics (Wheeler, Misner, Brill)**
- **Fundamental Two-Body Problem in GR (Einstein, DeWitt)**
- **Cosmic Censorship, Can a BH Break a BH (Penrose)?**
- **Powerful Source of Grav. Radn. (Thorne, Hawking)?**
- **Supercomputers Were Getting Fast Enough**
- **I Was Getting Married and I Needed a Ph.D…**



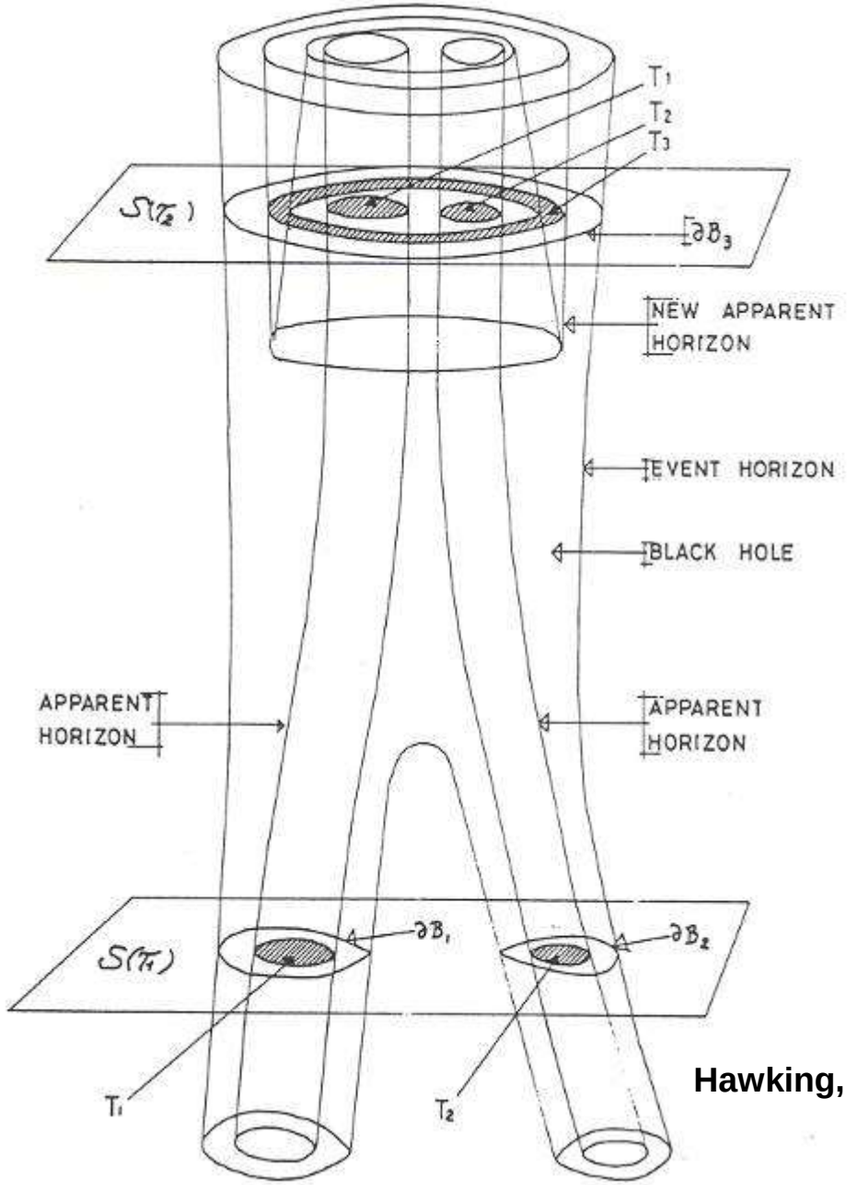


What is the End State of Two Colliding Black Holes?

"These considerations have very little to say about large perturbations, however. We might, for example, envisage two comparable black holes spiraling into one another. Have we any reason, other than wishful thinking, to believe that a black hole will be formed, rather than a naked singularity?
Very little, I feel; it is really a completely open question."

--Roger Penrose,
6th Texas Symposium on Relativistic Astrophysics, p. 131 (1973)

Expected Behavior of Event Horizon and Apparent Horizons



This Was the Status of Knowledge As I Started to Work on the 2BH Collision In 1972…

Hawking, Les Houches Lectures, p. 597 (1972)

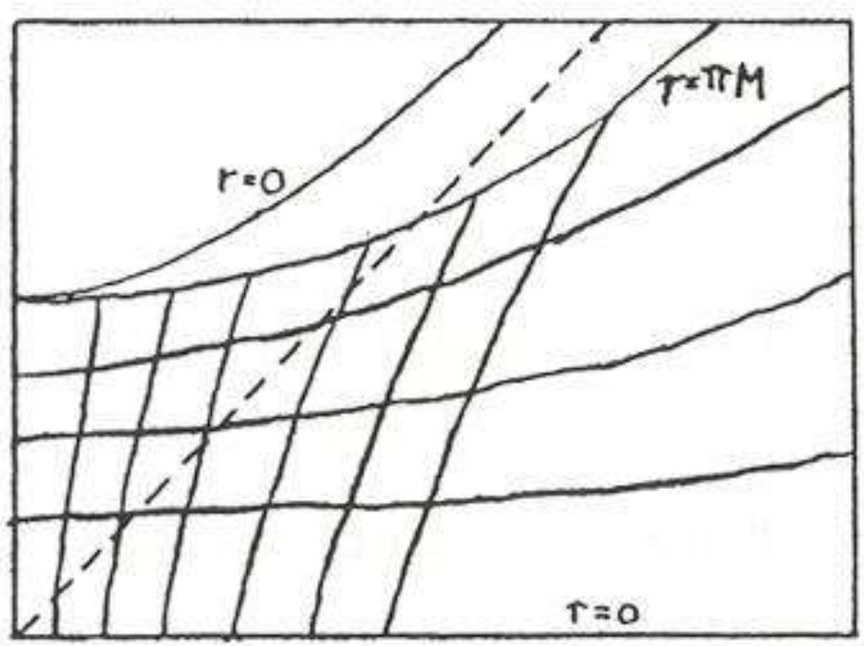
Maximal Slicing and the Two Black Hole Problem

- **1944 Lichnerowicz**
  - Maximal Slicing as a Coord. Condition “Like Incompressible Fluid”
- **1958-67 Dirac, Misner, Komar, DeWitt**
  - Maximal as Gauge Condition for Quantum Gravity or Energy Formula
- **1964 Hahn and Lindquist**
  - Geodesic Slicing of Two Einstein-Rosen Throats
- **1972 Cadez**
  - Maximal Slicing of Two Black Holes with Anti-Symmetric BCs
- **1973 Estabrook, Wahlquist, Christensen, DeWitt, Smarr, Tsiang; Reinhart**
  - Maximal Slicing of Schwarzschild/Kruskal-Numerically and Exact
- **1977 Smarr and Eppley**
  - Maximal Slicing of Two Black Holes

Results in a Coupled Elliptical/Hyperbolic System of PDEs

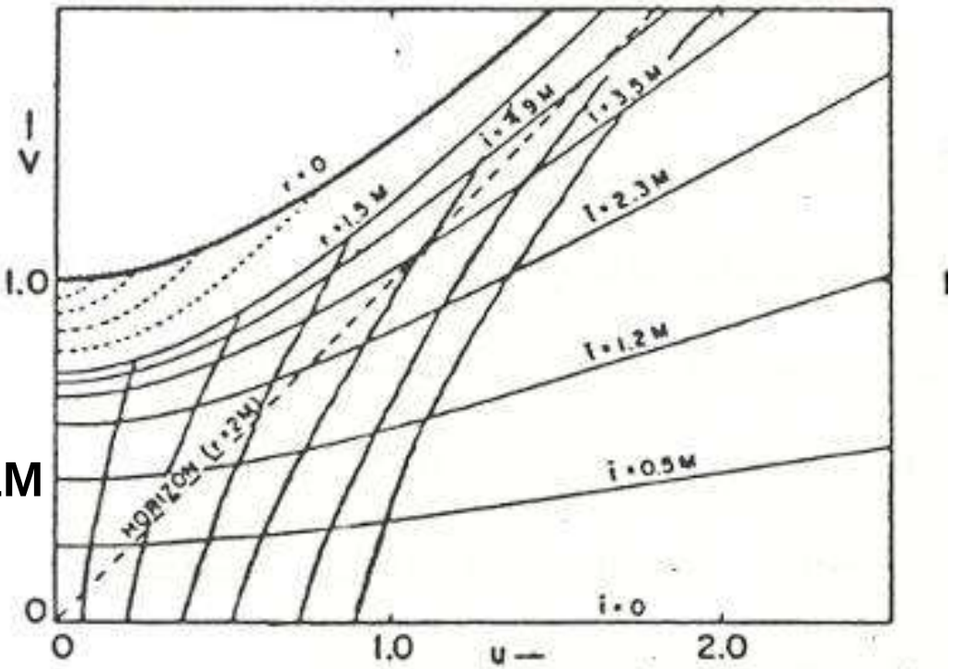
Geodesic vs. Maximal Slicings of One Black Hole: Maximal Slicing Avoids The Singularity

$\tau_{proper}=\pi M$



Geodesic Normal

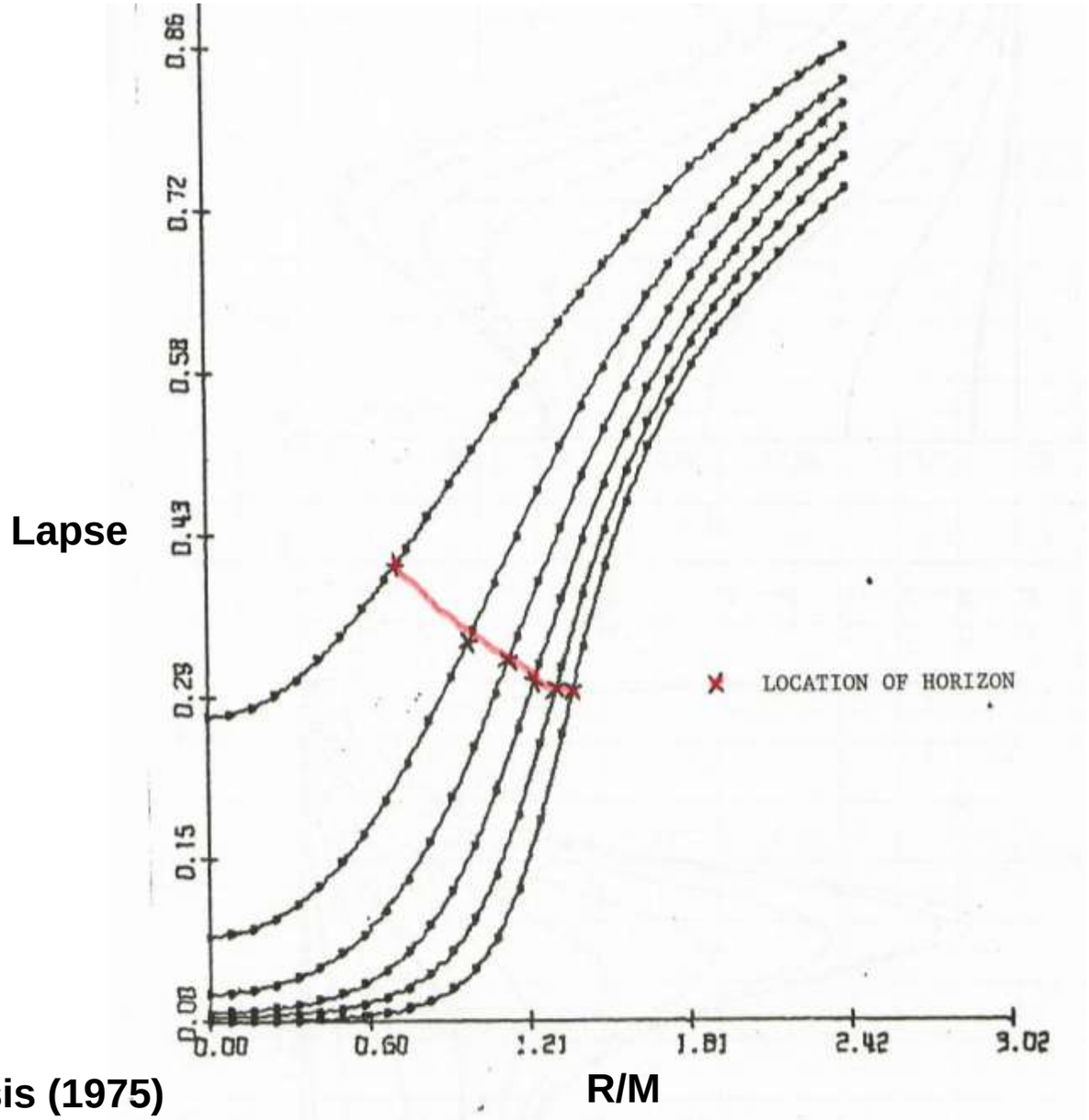
$\tau_{proper}=1.91M$



Maximal $\dot{\alpha}\neq 0$ $\beta_r=0$

Smarr, Ph.D. Thesis (1975), p.126

Collapse of the Lapse
In 1D Maximal Slicing of One Black Hole



Source: Ken Eppley PhD Thesis (1975)

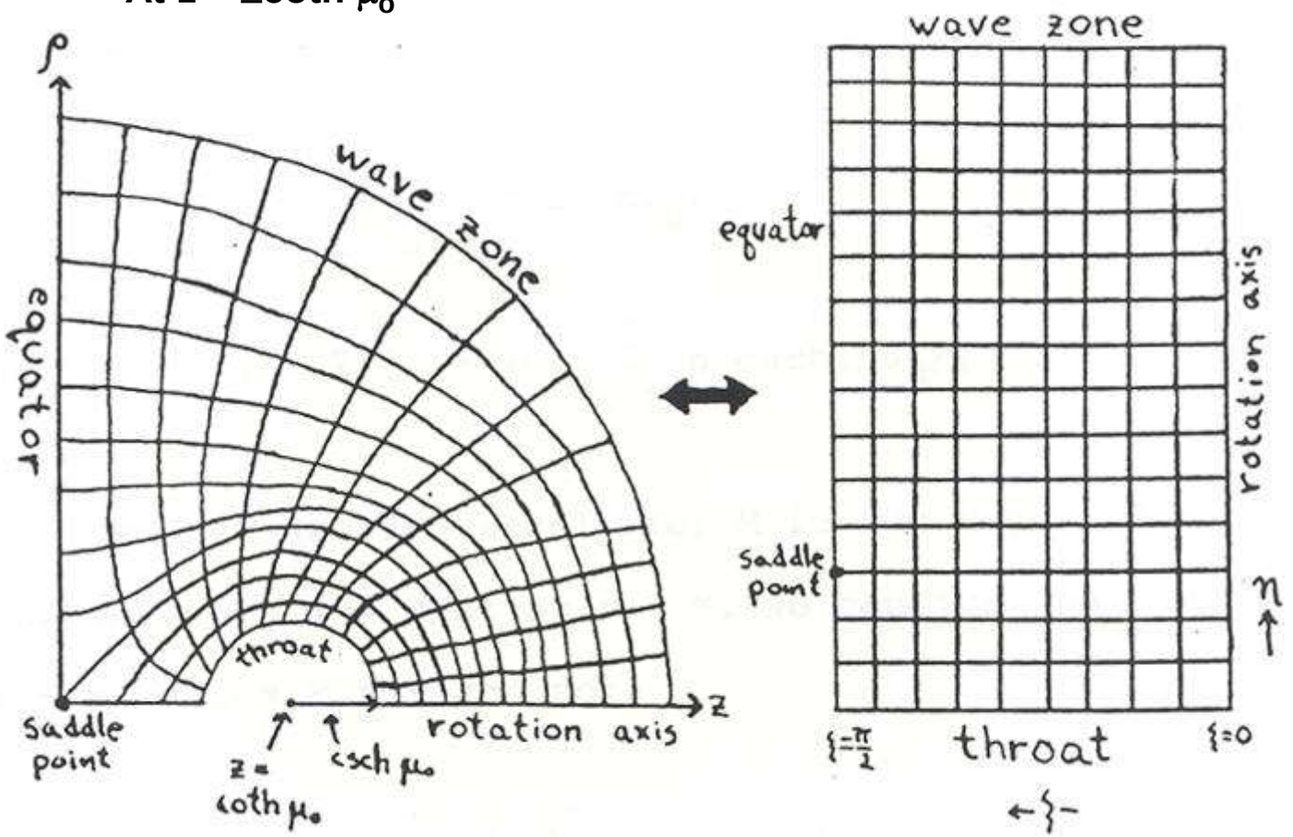
Shift from Misner Coordinates to Cadez Coordinates: Mapping to Cylindrical Coordinates

$$\eta + i\xi = \chi(z + i\rho) = \ell n \zeta$$

$$\eta = \ell nR = \tfrac{1}{2}\ell n\,(\rho^2 + z^2)$$

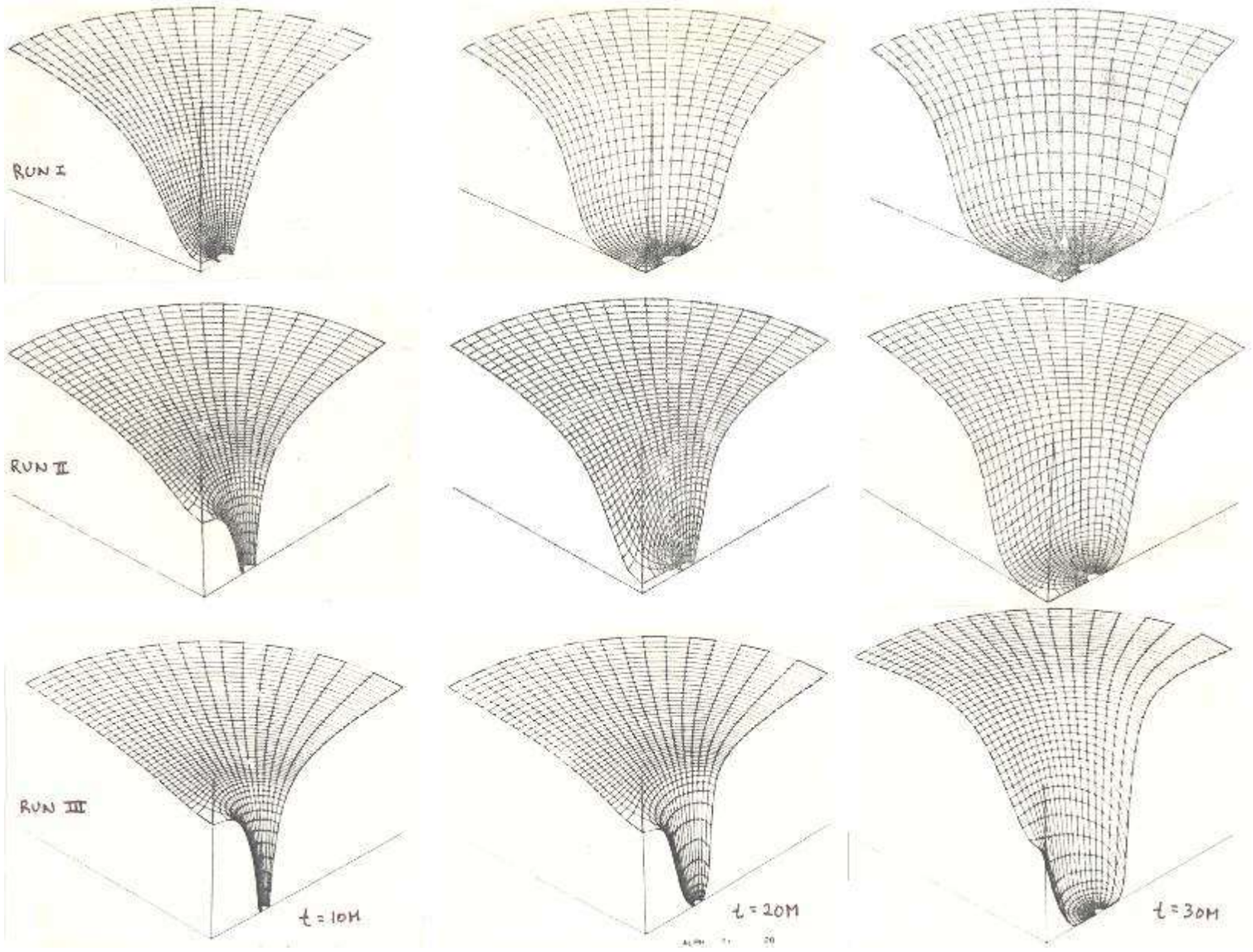
$$\xi = \theta = \tan^{-1}\left(\frac{\rho}{z}\right)$$

Coordinates are Field Lines and Equipotentials for Two Equal Charges At $z = \pm\coth \mu_o$



Smarr, Cadez, DeWitt, & Eppley Phys. Rev. D14, 2448 (1976)

Collapse of Lapse
for The Three Black Hole Collision Runs



Run I μ_o=2.00 **(Already Merged)**

Run II μ_o=2.75 **(Near Collision)**

Run III μ_o=3.25 **(Far Collision)**

Eppley and Smarr, Research Notes (1977)

Isometric Embedding of Two Black Hole Collision 3-Space

μ_o=5.0

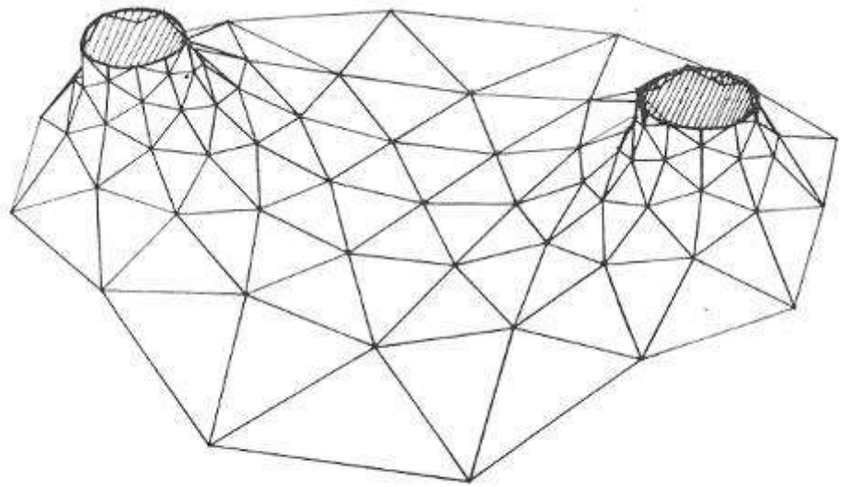
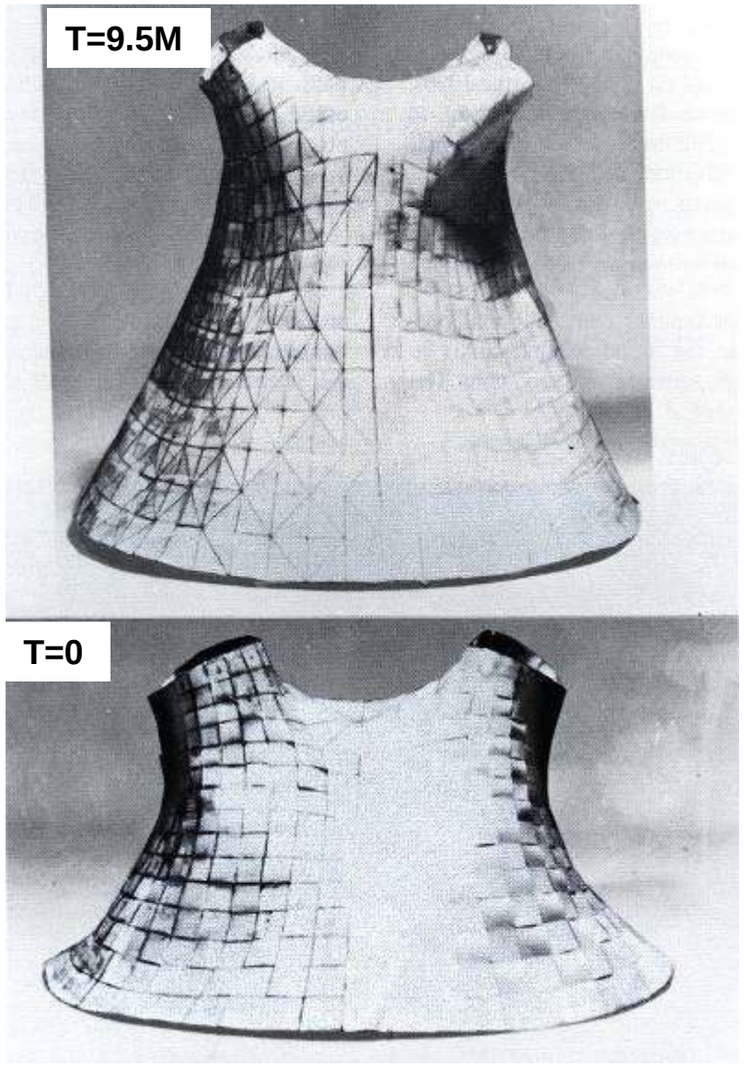


FIG. 2. The geometry of the time symmetric slice of a spacetime with two black holes. In the construction of this figure, m_1 was taken equal to m_2 and $a/m_1 = 18.8$ (see Eq. (10)).

Some remarks on the two-body-problem in geometrodynamics

Andrej Čadež *

Cadez, Ann. Physics, 91 p. 62 (1975)



μ_o=2.0

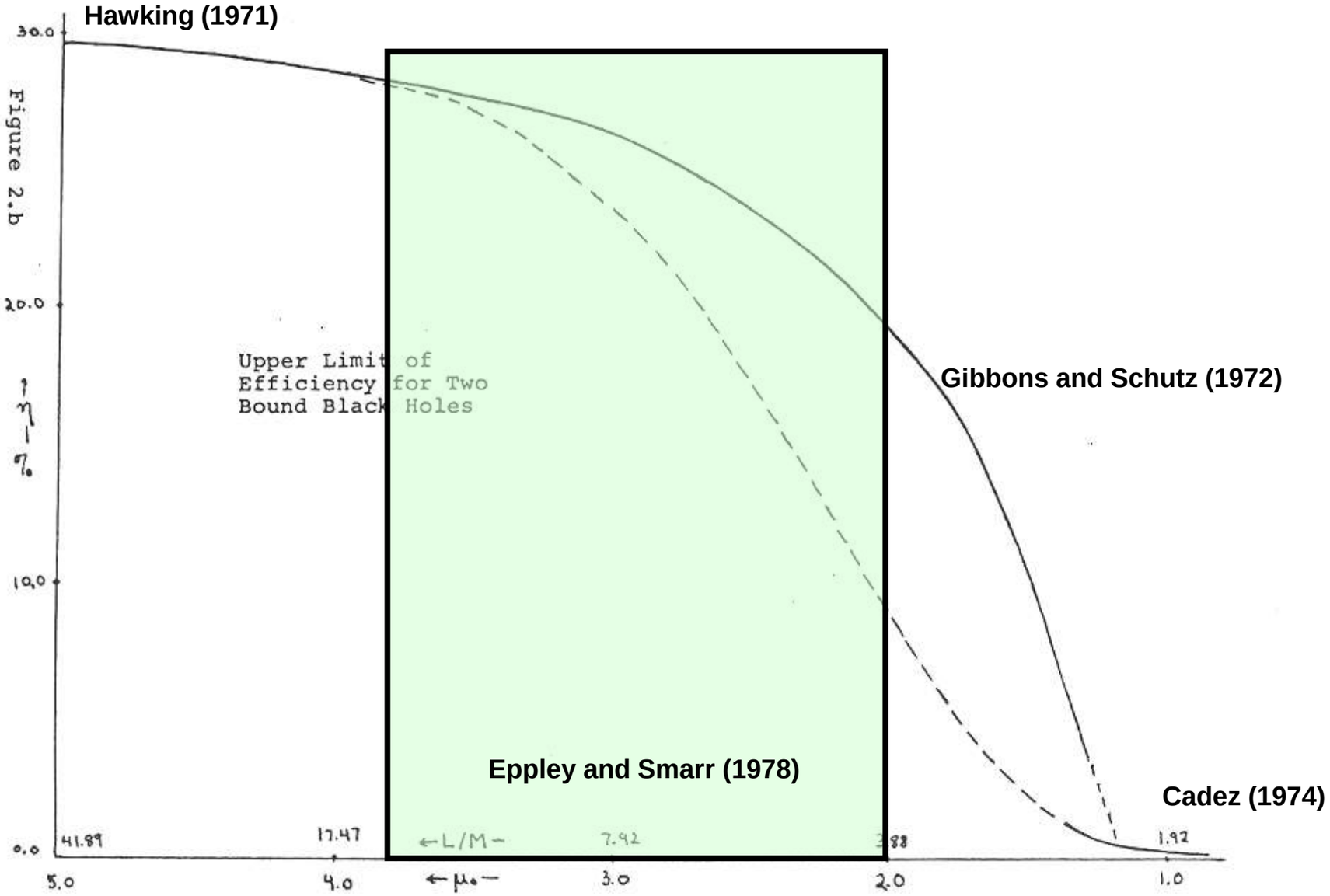
Smarr, 8th Texas Symposium, p. 597 (1977)

Eppley, Ph.D. Thesis (1975), p.239

Gravitational Radiation From Colliding Black Holes

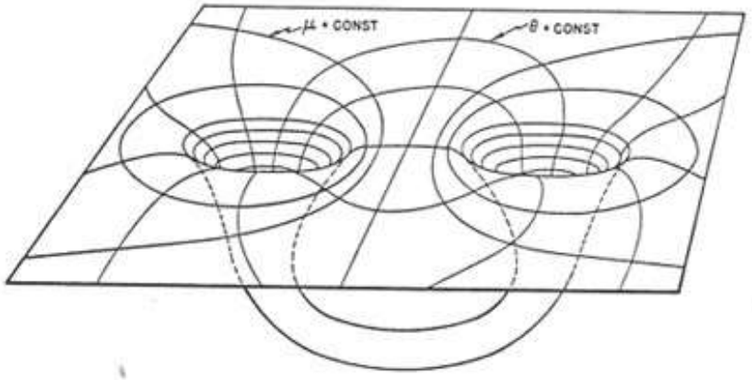
- 1959 Brill, Bondi, Weber, Wheeler, Araki
  - Time Symmetric Gravitational Waves
- 1971 Press
  - Existence of Normal Modes of Black Holes
- 1971 Davis, Ruffini, Press, Price
  - Radn. From Particle Falling Radially Into Black Hole
- 1971 Hawking
  - Area Theorem Upper Limits on Grav. Radn. From 2BHs
- 1972 Gibbons, Schutz, Cadez
  - Area Theorem Uppers Limits for Two Bound Black Holes
- 1977 Teukolsky
  - Linearized Analytic Solution for Time Symmetric Waves
- 1978 Eppley and Smarr
  - Wave Forms and Amplitudes for Different 2BH Initial Data

Hawking Area Theorem Upper Limits to Grav. Radn. Efficiency from Bound 2BH Collision

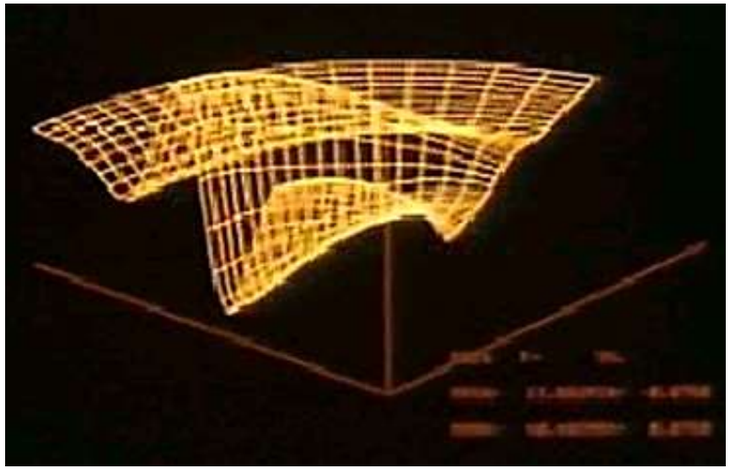


Smarr, Ph.D. Thesis (1975), p.135

Supercomputer Speed Had Increased Since the First Numerical Attempt at the 2BH Collision Problem



1963
Hahn & Lindquist
IBM 7090
One Processor
Each 0.2 Mflops
3 Hours

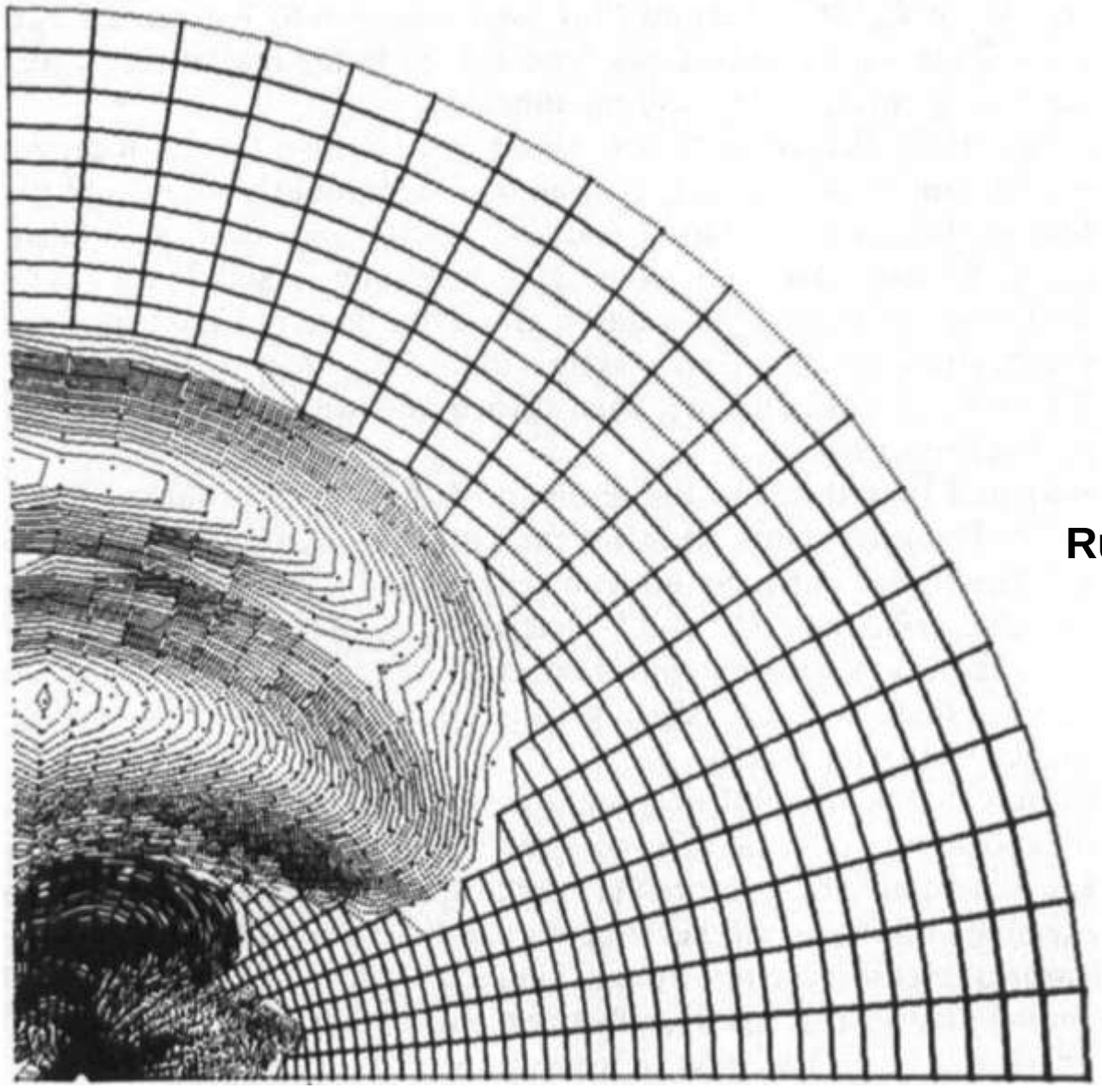


1977
Eppley & Smarr
CDC 7600
One Processor
Each 35 Mflops
5 Hours

300X

An Early View of the Quadrupolar Gravitational Radiation Produced by the Head-On Collision of Two Black Holes

Contour Plot of the Radial Component of the Bel-Robinson Vector T=20M



Run II μ_o=2.75 (Near Collision)

Larry Smarr, “Spacetimes Generated by Computers: Black Holes With Gravitational Radiation,” Annals New York Academy of Sciences v. 302, p.592 (1977)

Comparison of Two Black Hole Collision Waveform and the DRPP Perturbation Waveform Indicated Ringing Dominated

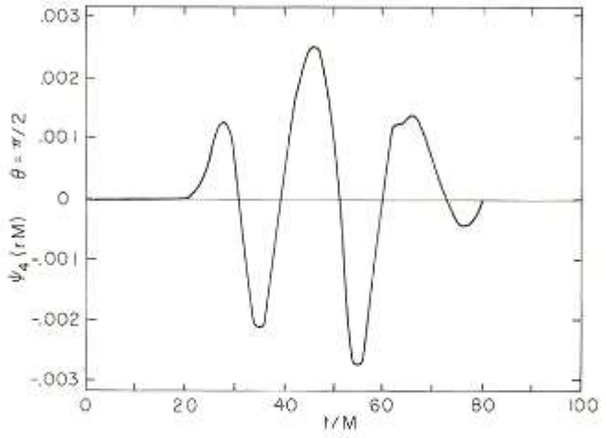


Figure 3. The curvature ψ_4·rM in the equatorial plane crossing the 2-sphere at r = 25M as a function of time. This is for the two black hole collision Run II.

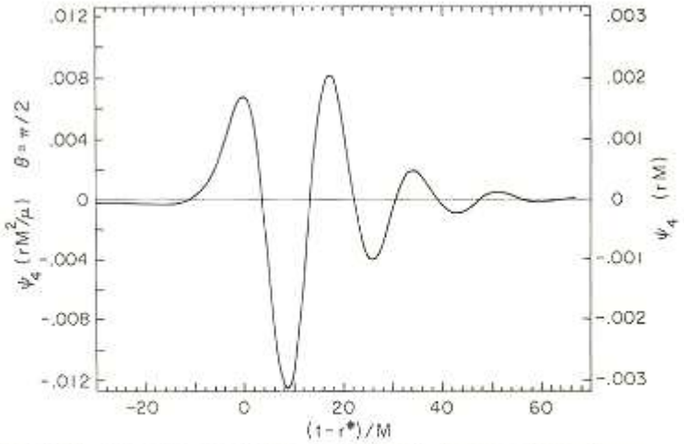
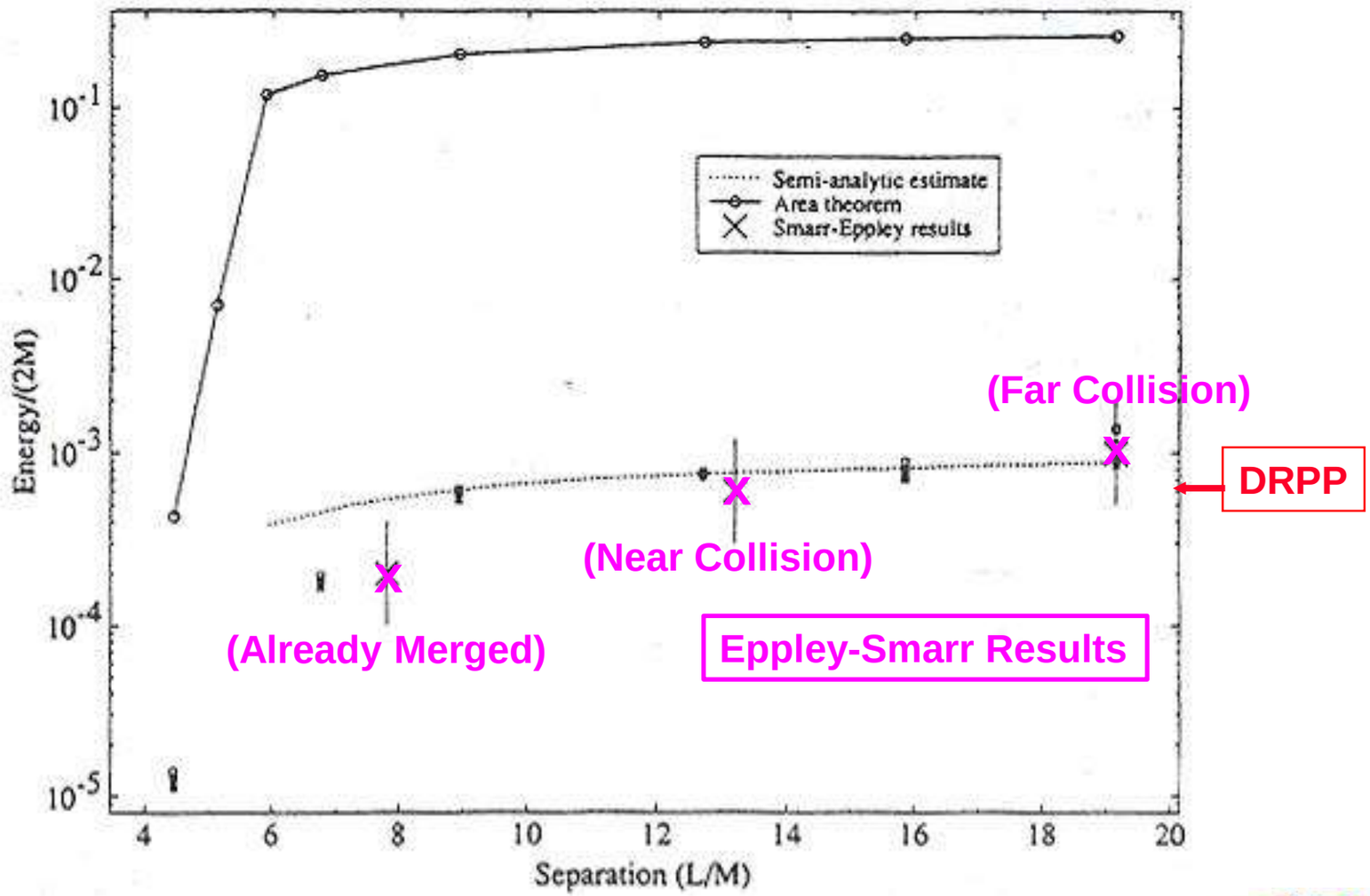


Figure 4. The same quantity as in Figure 3 except from the perturbation calculation of a particle of mass μ falling into a black hole of mass M. The abscissa is retarded time. The vertical scales are explained in the text. Only the quadrupole contribution is shown here.

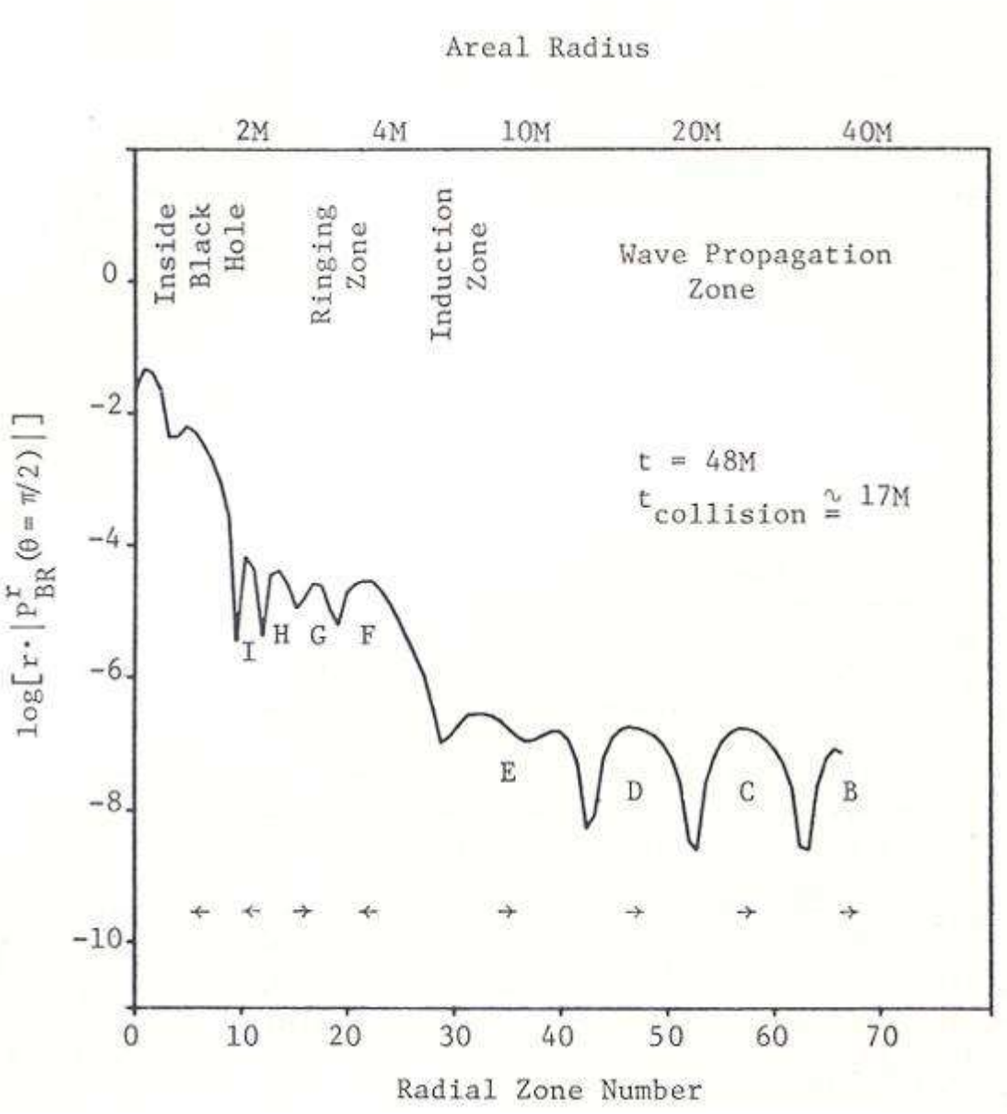
Smarr, Sources of Grav. Radn (1978), p.268



Anninos, Hobill, Seidel, Smarr, Suen, Phys. Rev. Lett., 71, p. 2854 (1993)

Numerical Relativity Reveals Wave Formation in Ringing Region

Log (Areal Radius r^2 x the Bel-Robinson Vector in the Equatorial Plane)



Run II μ_o=2.75 (Near Collision)

Smarr, Sources of Grav. Radn (1978), p.270

The End of The First Sixty Years of the 2 Black Hole Problem - A Bookend to the Chapel Hill Conference 30 Years Earlier

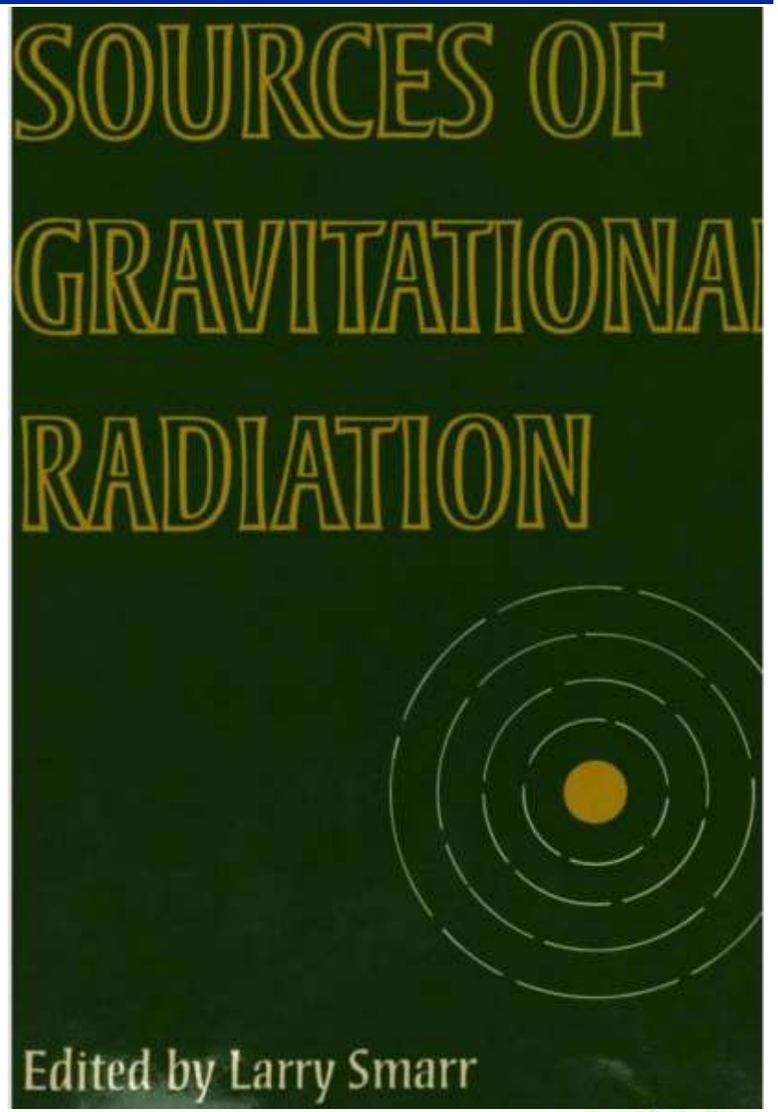
Sources of Gravitational Radiation

PROCEEDINGS OF THE BATTELLE SEATTLE WORKSHOP

July 24 - August 4, 1978

edited by

LARRY L. SMARR



Workshop Board of Advisors:
Bryce DeWitt, Frank Estabrook, Charles Misner, Jerry Ostriker, Bill Press, David Schramm, Kip Thorne, Rai Weiss, John Wheeler, Jim Wilson

Workshop Organizers:
Larry Smarr, Doug Eardley, Saul Teukolsky, Jim York

Local Organizers:
Jim Bardeen, P.C. Peters, Battelle Staff

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Forty Years After the 1978 Seattle Battelle Workshop
Two Authors Receive the Nobel Prize



The Nobel Prize in Physics 2017

Rainer Weiss, Barry C. Barish, Kip S. Thorne

GRAVITATIONAL WAVE DETECTORS

Gravitational Radiation - The Status of the Experiments and Prospects for the Future
R. Weiss

Gravitational Radiation Detection with Spacecraft Doppler Tracking: Limiting Sensitivities and Prospective Missions
F.B. Estabrook, R.W. Hellings, H.D. Wahlquist and R.S. Wolff

The Quantum Limit for Gravitational-Wave Detectors and Methods of Circumventing It
K.S. Thorne, C.M. Caves, V.D. Sandberg and M. Zimmermann





Historical Summary by Kip Thorne
From His Nobel Prize Lecture in 2018

Brief History of Simulations

- Late 1950s: Motivation - Geometrodynamics



John Wheeler

- 1958-64: Foundations



Charles Misner



Susan Hahn

Richard Lindquist

- 1970-78: Head-on collision of two black holes



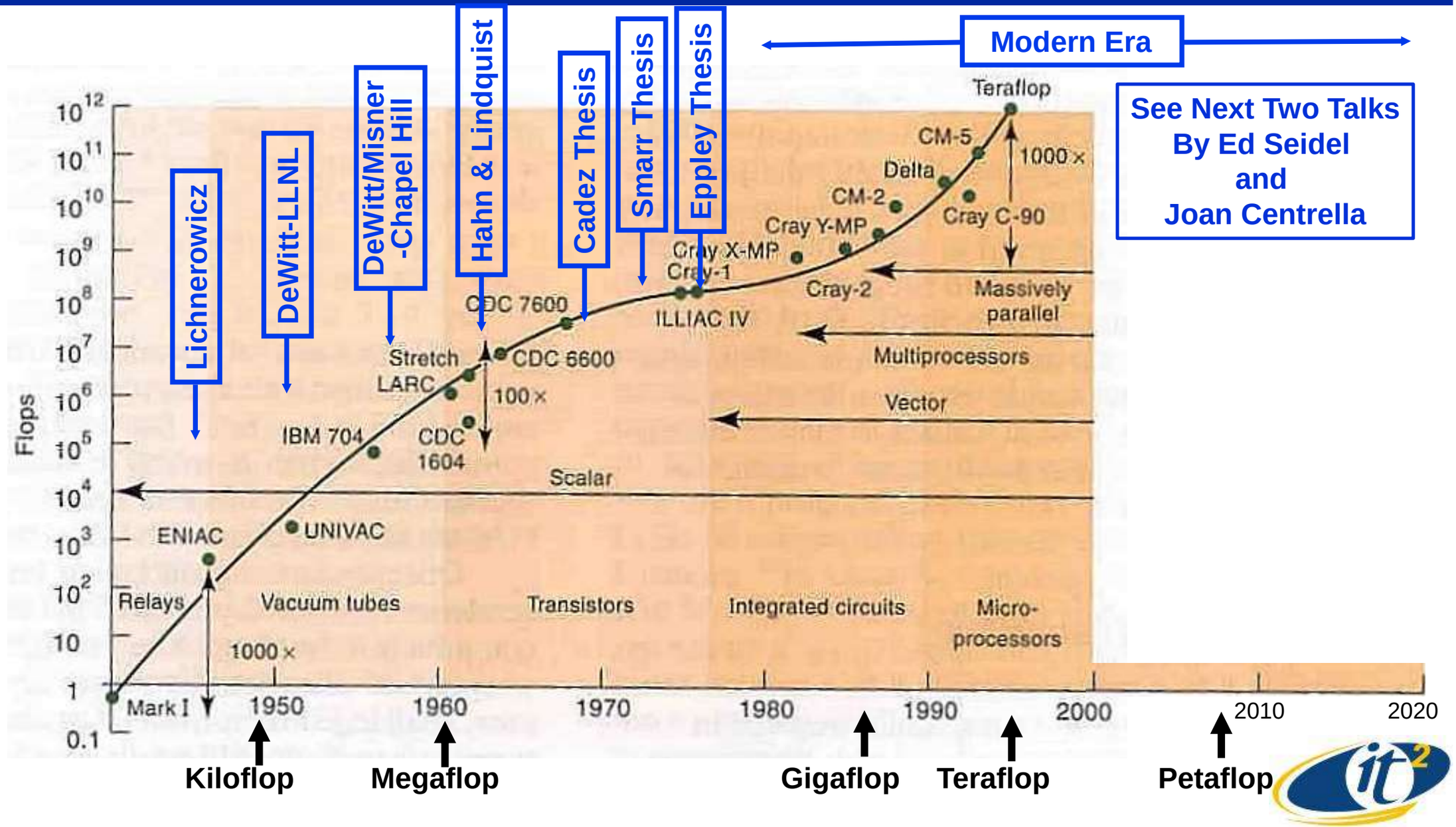
Bryce DeWitt



Larry Smarr

Kenneth Eppley

The Numerical Two Black Hole Collision Problem Spans the Digital Computer Era



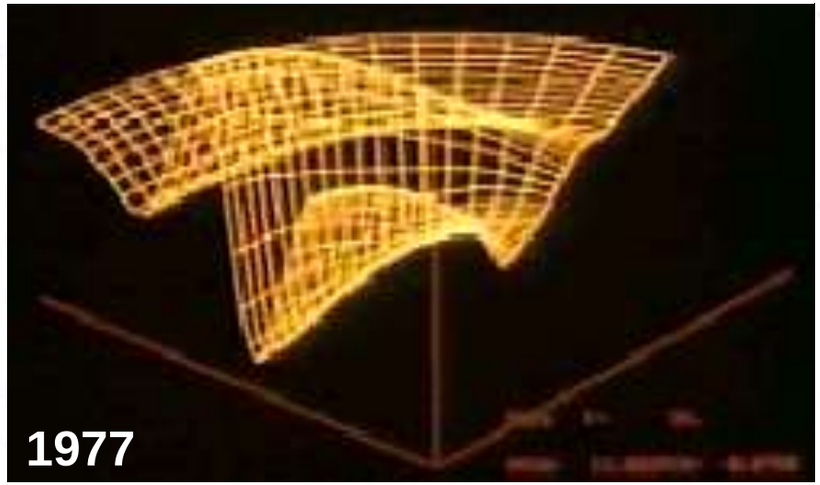
Forty Years of Computing Gravitational Waves From Colliding Black Holes – One Billion Times Increase in Supercomputer Speed!

THE STRUCTURE OF GENERAL RELATIVITY WITH A NUMERICAL ILLUSTRATION: THE COLLISION OF TWO BLACK HOLES

by

Larry Lee Smarr, B.A., M.S.

L. Smarr and K. Eppley
Gravitational Radiation Computed from an Axisymmetric Black Hole Collision



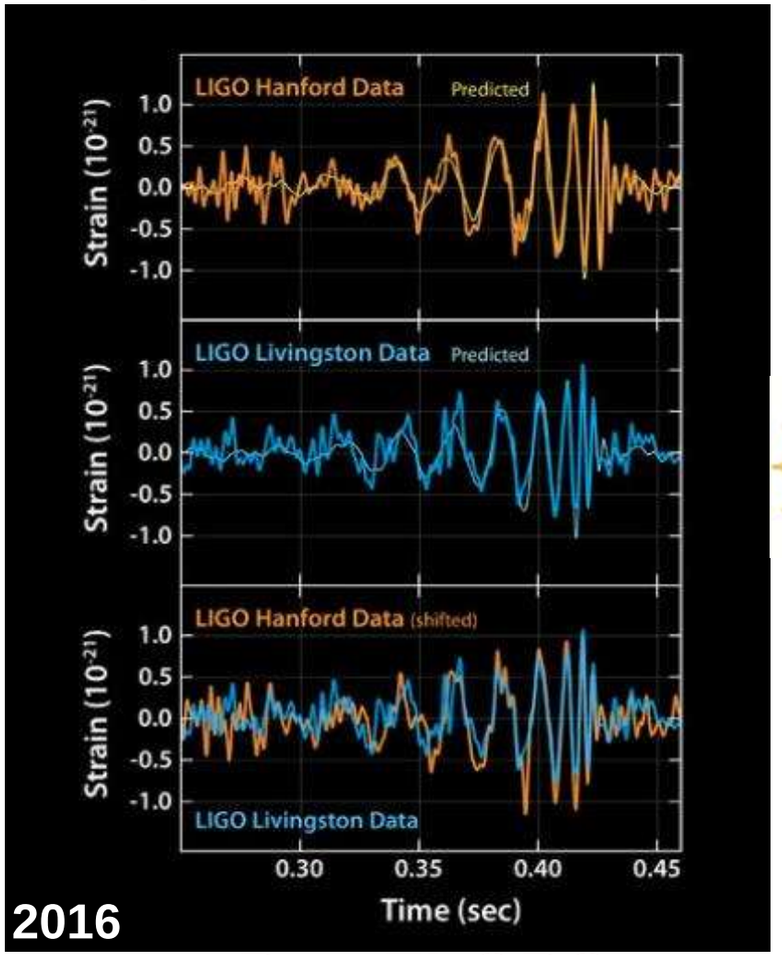
MegaFLOPS

THE UNIVERSITY OF TEXAS AT AUSTIN

January 1975

40 Years

LIGO Consortium
Spiral Black Hole Collision



PetaFLOPS

Holst, et al. Bull. Amer. Math. Soc 53, 513-554 (1916)