The First 50 Years of the Two Black Hole Collision Problem: 1935 to 1985

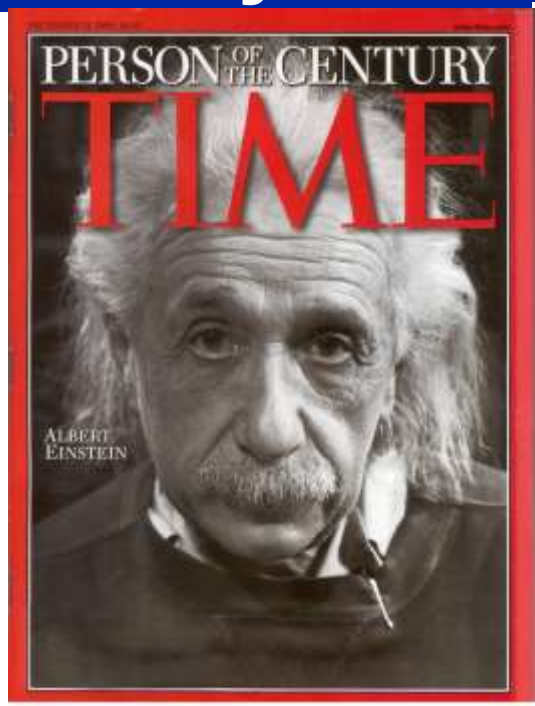
Invited Talk at the UCSB Institute of Theoretical Physics Miniprogram on Colliding Black Holes:

Mathematical Issues in Numerical Relativity,

Santa Barbara, CA

January 10, 2000

The Problem of the Century Posed by the Person of the Century

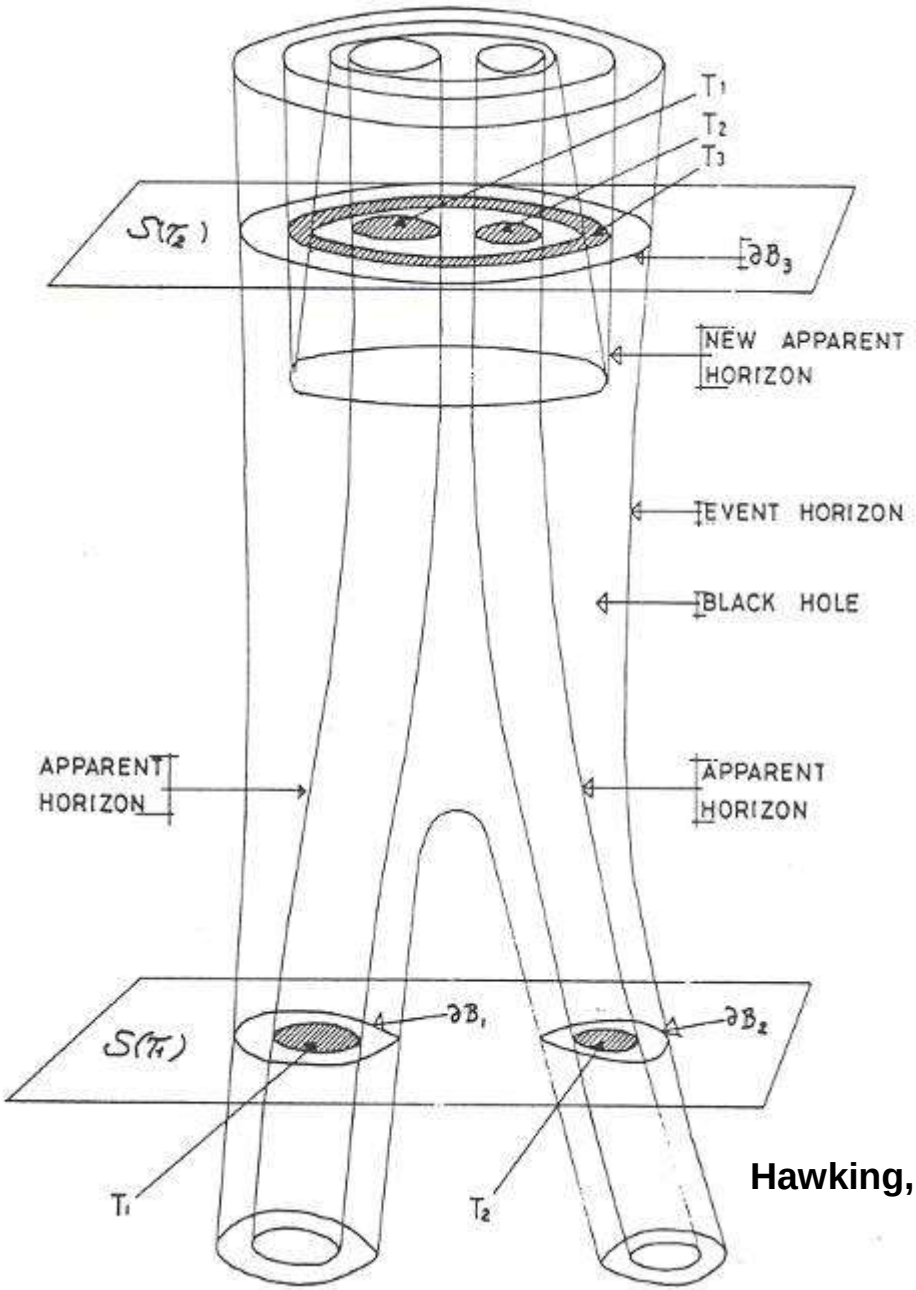


- 1910s-General Theory; Schwarzschild
- 1920s-Equation of Motion Posed
- 1930s-Two Body Problem Posed
- 1940s-Cauchy Problem Posed
- 1950s-Numerical Relativity Conceived
- 1960s-Geometrodynamics; First Numerical Attempts
- 1970s-Head-On Spacetime Roughed Out
- 1980s-Numerical Relativity Becomes a Field
- 1990s-Head-On Nailed; 3D Dynamics Begin
- 2000s-3D Dynamics Nailed; Grav. Wave Astronomy

Why Did I Attack the Two Black Hole Problem in 1972?

- Explore Geometrodynamics (Wheeler, Misner, Brill)
- Fundamental Two-Body Problem in GR (Einstein, DeWitt)
- Cosmic Censorship, Can a BH Break a BH (Penrose)?
- Powerful Source of Grav. Radn. (Thorne, Hawking)?
- Supercomputers Were Getting Fast Enough
- I Needed a Ph.D…

Behavior of Event Horizon and Apparent Horizons



1963 Kerr
1968 “Black Hole”

This Was the Status of Knowledge
As I Started to Work on the 2BH Collision
In 1972…

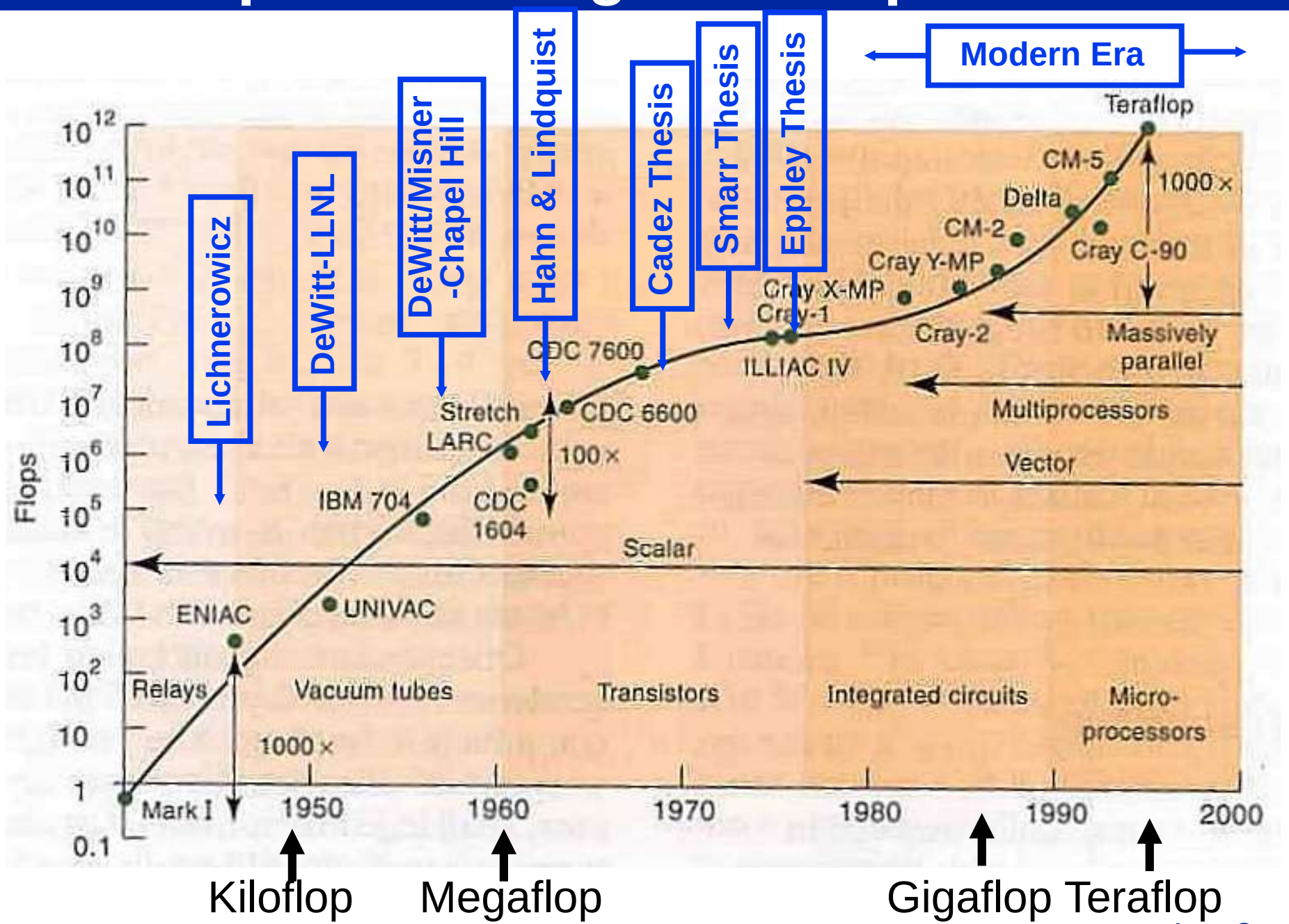
Hawking, Les Houches Lectures, p. 597 (1972)

What is the End State of Two Colliding Black Holes?

"These considerations have very little to say about large perturbations, however. We might, for example, envisage two comparable black holes spiraling into one another. Have we any reason, other than wishful thinking, to believe that a black hole will be formed, rather than a naked singularity? Very little, I feel; it is really a completely open question."

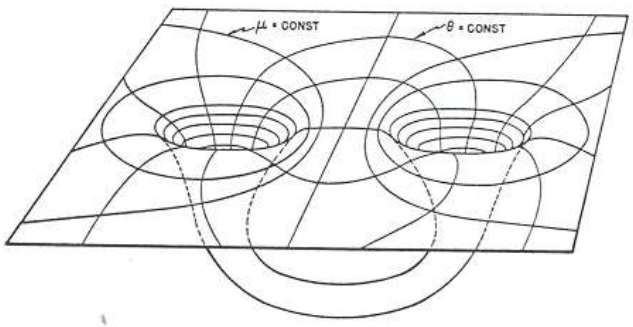
--Roger Penrose, 6th Texas Symposium on Relativistic Astrophysics, p. 131 (1973)

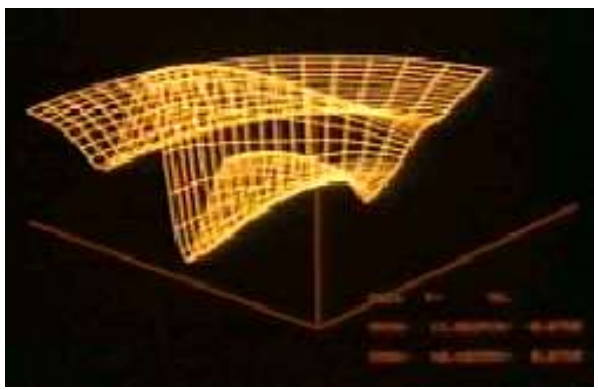
The Numerical Two Black Hole Problem Spans the Digital Computer Era

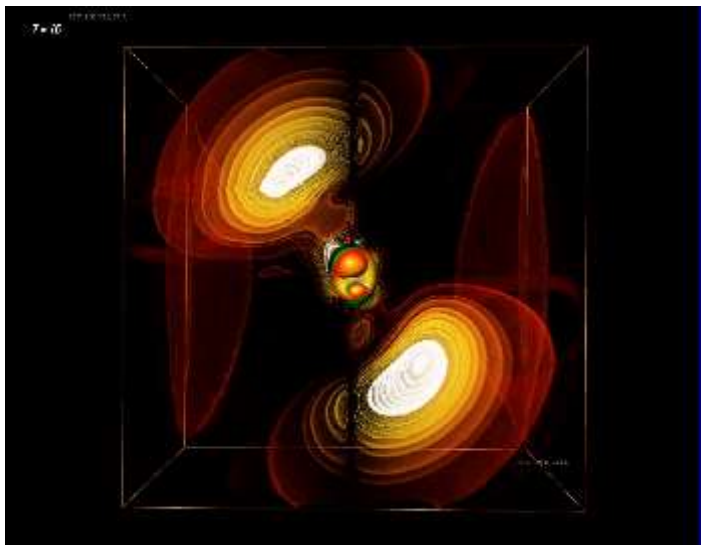


Relative Amount of Floating Point Operations for Three Epochs of the 2BH Collision Problem

9,000,000X







1963	1977	1999
Hahn & Lindquist	Eppley & Smarr	Seidel & Suen, et al.
IBM 7090	CDC 7600	SGI Origin
One Processor	One Processor	256 Processors
Each 0.2 Mflops	Each 35 Mflops	Each 500 Mflops
3 Hours	5 Hours	40 Hours

300X **30,000X**

The Cauchy Evolution of Initial Data

- 1944 Lichnerowicz
  - 3+1 Decomposition, Idea of Numerical Integration
- 1956 Choquet-Bruhat
  - Formalizes Cauchy Problem
- 1957 DeWitt, Misner
  - Concept of Numerical Relativity
- 1959 Wheeler, Misner
  - Geometrodynamics and Superspace
- 1961 Arnowitt, Deser, & Misner
  - Canonical Decomposition
- 1970 Geroch
  - Domain of Dependence
- 1971 York
  - Initial Value Problem
- 1978 Smarr and York
  - Spacetime Engineering

André Lichnerowicz
"L'intégration des Équations de la Gravitation Relativiste et le Problème des n Corps"

- **Sets up Cauchy Problem in 3+1 Form ($\partial_t K^i_j$=…)**
- **Studies Minimal Surfaces and Finds**
  - **K=0 Means Minimal if Shift Vector is Zero**
  - **Elliptic Lapse Equation**
  - **Normal Congruence Behaves Like Irrotational Incompressible Fluid**
- **Finds Elliptic Eqn. for 3-Metric Conformal Factor**
- **Sets Up n-Body Problem with Matter**
  - **Time Symmetric Initial Data for Conformally Flat 3-Space**
  - **Geodesic Normal Gauge for Evolution**
  - **Uses Matter Instead of non-Euclidean Topology as Body Models**
  - **Solves for Conformal Factor and Exhibits Interaction Energy**
- **"A de telles donnés correspondra une solution *rigoureuse* de ce problème, dont l'évolution dans le temps sera régie par les équations et pourra être obtenue par une intégration numérique de ces équations."**

Journal de mathematiques pures et appliques **23**, 37 (1944)

Chapel Hill Conference on the Role of Gravitation in Physics 1957

- **Bryce DeWitt asked if the Cauchy problem is now understood sufficiently to be put on an electronic computer for actual calculation.**
- **Charles Misner answered that he had computed initial data for two Einstein-Rosen throats that "can be interpreted as two particles which are non-singular… These partial differential equations, although very difficult, can then in principle be put on a computer."**
- **Misner thinks that one can now give initial conditions so that one would expect to get gravitational radiation, and computers could be used for this.**
- **DeWitt pointed out some difficulties encountered in highspeed [hydro] computational techniques. "Similar problems would arise in applying computers to gravitational radiation since you don't want the radiation to move quickly out of the range of your computer."**

Wright Air Development Center Technical Report 57-216 (1957)

The First Crisp Definition of Numerical Relativity

- **Misner Summarizes—**
  - **"First we assume that have a computing machine better than anything we have now, and many programmers and a lot of money, and you want to look at a nice pretty solution of the Einstein equations. The computer wants to know from you what are the values of $g_{\mu\nu}$ and $\partial_t g_{\mu\nu}$ at some initial surface. Mme. Foures has told us that to get these initial conditions you must specify something else and hand over that problem, the problem of the initial values, to a smaller computer first, before you start on what Lichnerowicz called the evolutionary problem. The small computer would prepare the initial conditions for the big one. Then the theory, while not guaranteeing solutions for the whole future, says that it will be some finite time before anything blows up."**

Note Supercomputers Are Still Using Vacuum Tubes at This Time!

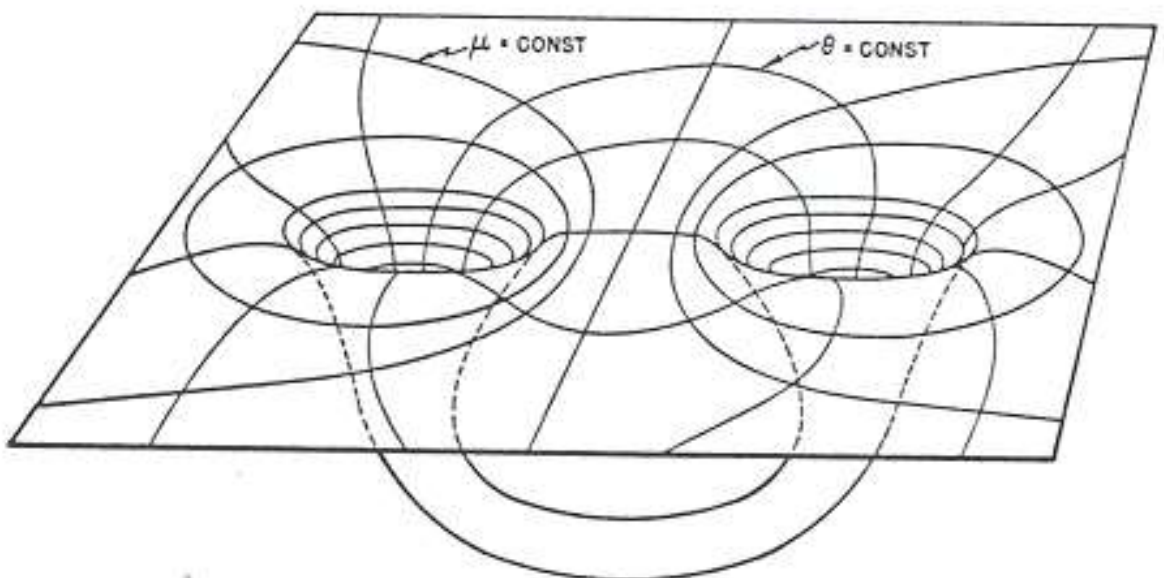
Wright Air Development Center Technical Report 57-216 (1957)

Geometrodynamics of Wormholes
"Mass Without Mass"

"Geometrodynamics and the Problem of Motion"

"The evolution in time of the wormhole 3-geometry thus specified can be found in the beginning by power series expansion and thereafter by electronic computation. The intrinsic geometry of the resulting 4-space is completely determinate, regardless of the freedom of choice that is open as to the coordinate system to be used to describe that geometry. This geometry contains within itself the story as the change of the distance L between the throats with time and the generation of gravitational waves by the two equal masses as they are accelerated towards each other."

--John Archibald Wheeler, Rev. Mod. Phys. 33, 70 (1961)

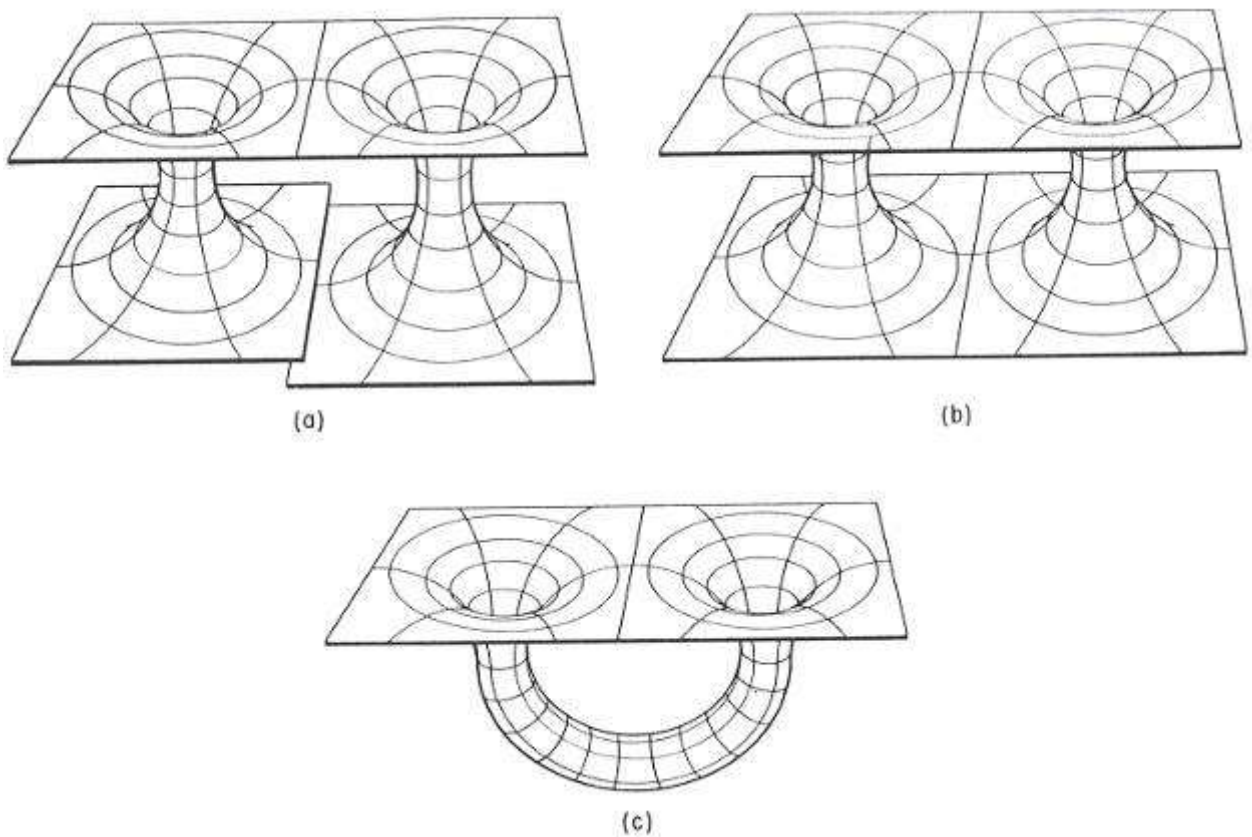


Misner, Phys. Rev., 118, p. 1110 (1960)

Two Black Hole Initial Data

- 1935 Einstein and Rosen
  - Particles Represented by “Bridges” Connecting “Sheets”
- 1944 Lichnerowicz
  - Matter as n Bodies
- 1960 Misner
  - Wormhole Initial Conditions
- 1963 Misner
  - The Method of Images in Geometrostatics
- 1963 Lindquist
  - Initial Value Problem on Einstein-Rosen Manifolds
- 1963 Brill and Lindquist
  - Interaction Energy
- 1970 Cadez
  - Bispherical Coordinates
- 1984 Bowen, Rauber, York, Piran, Cook
  - General 2BH Initial Data

The Different Topologies for the Two Body Problem



Hahn and Lindquist, Ann.Phys., 29, p. 307 (1964)

Hahn and Lindquist
"The Two Body Problem in Geometrodynamics"

- Conceptually Studying Causality and Area of Throats
- Black Hole is not a Term until Four Years Later
- Used Misner Coordinates
  - Good Near Throats
  - Terrible at Large Distances
  - Mesh Size 51x151
- Used Geodesic Normal Coordinates
- Initial Data of Black Holes "Almost Merged" (μ_o=1.6)
- Used IBM 7090 (~0.3 MFLOPS)
  - Integrated Very Short Time to Future (<0.3M)
- Proof of Principle that Numerical Relativity Worked

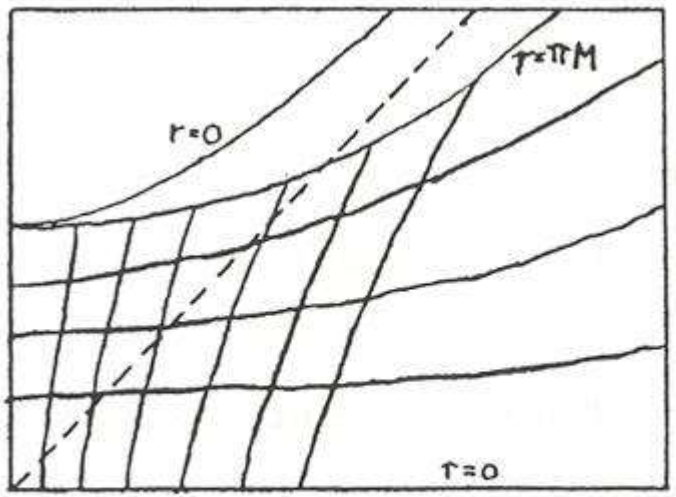
Hahn and Lindquist, Ann.Phys., 29, p. 304 (1964)

Maximal Slicing and the Two Black Hole Problem

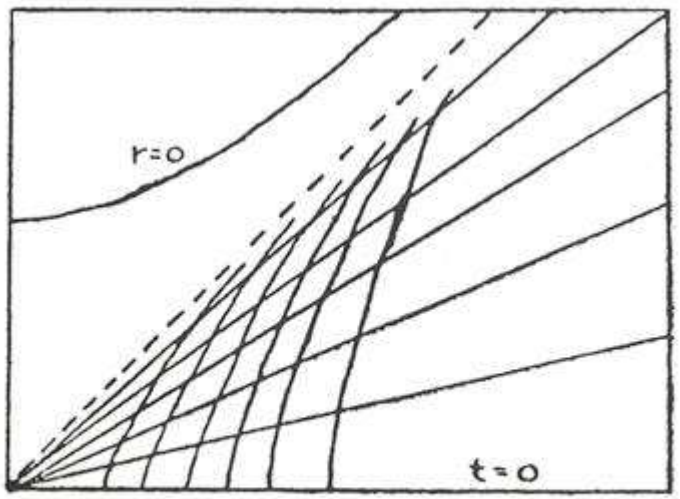
- 1944 Lichnerowicz
  - Maximal Slicing as a Coord. Condition "Like Incompressible Fluid"
- 1958-67 Dirac, Misner, Komar, DeWitt
  - Maximal as Gauge Condition for Quantum Gravity or Energy Formula
- 1964 Hahn and Lindquist
  - Geodesic Slicing of Two Einstein-Rosen Throats
- 1972 Cadez
  - Maximal Slicing of Two Black Holes with Anti-Symmetric BCs
- 1973 Estabrook, Wahlquist, Christensen, DeWitt, Smarr, Tsiang; Reinhart
  - Maximal Slicing of Schwarzschild/Kruskal-Numerically and Exact
- 1978 Smarr and Eppley
  - Maximal Slicing of Two Black Holes
- 1978 Smarr and York
  - Analytic Lapse Collapse Calculation
- 1979 Eardley & Smarr; Choquet-Bruhat; Marsden & Tipler; York
  - K=0 and K=Constant Singularity Avoidance Theorems

Slicings of One Black Hole

$\tau_{proper}=\pi M$

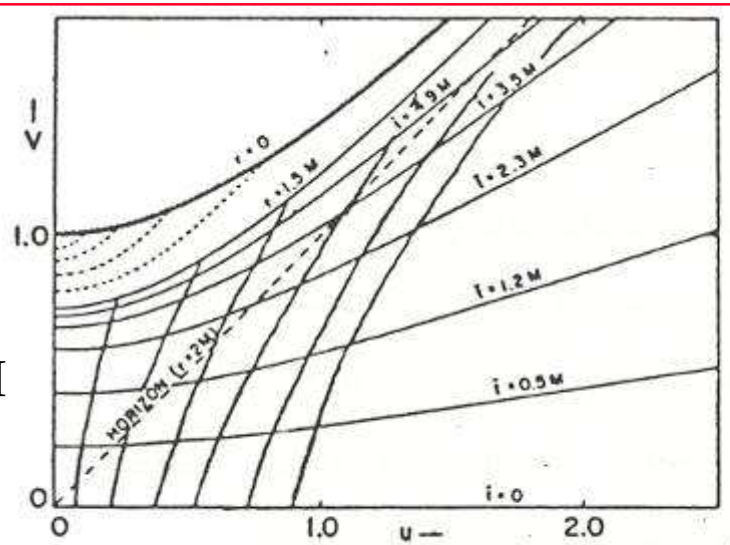


Geodesic Normal

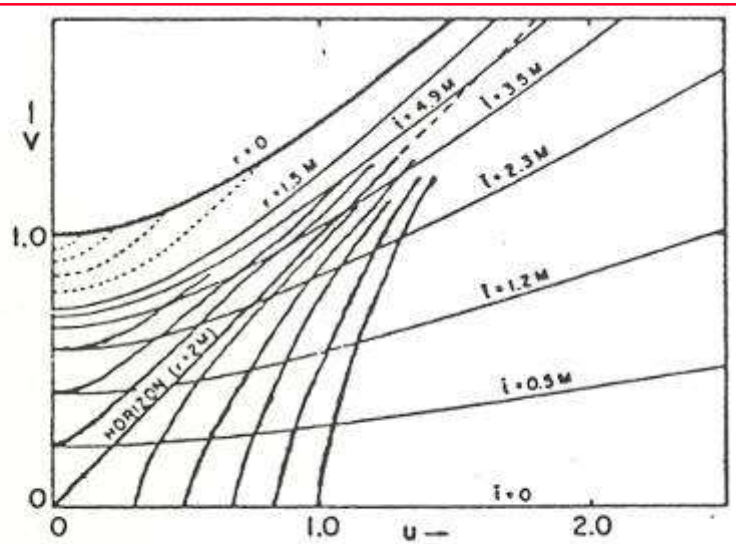


Maximal $\dot{\alpha} = 0$

$\tau_{proper}=1.91M$



Maximal $\dot{\alpha}\neq 0$ $\beta_r=0$



Maximal $\dot{\alpha}\neq 0$ $\beta_r\neq 0$

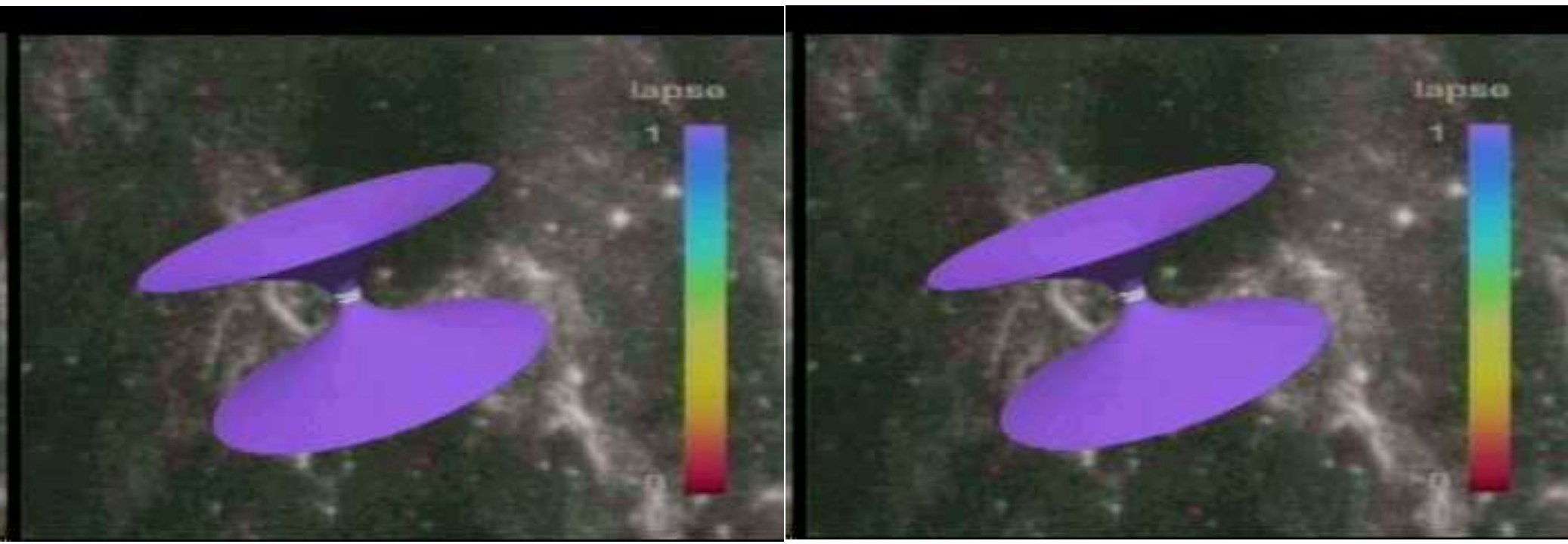
Smarr, York, Phys. Rev. 17, 2529 (1978)

Smarr, Ph.D. Thesis (1975), p.126

Geodesic versus Maximal Slicing of Schwarzschild-Kruskal Spacetime

Free Fall to Singularity

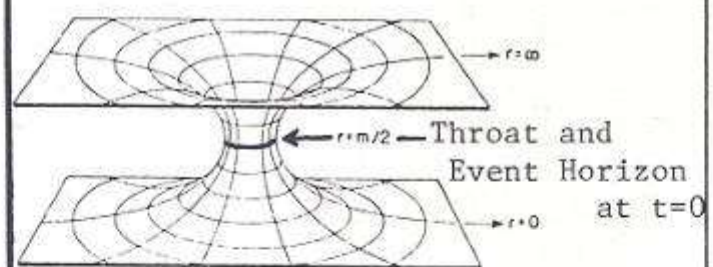
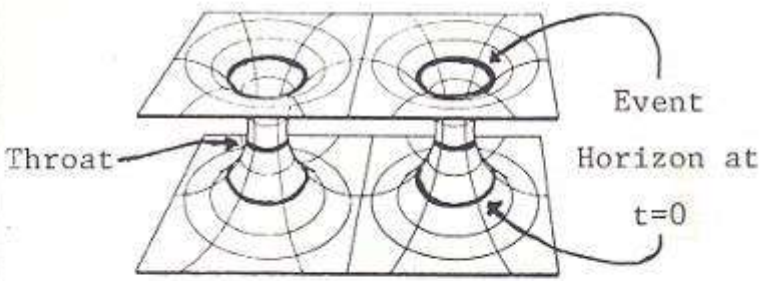
Singularity Avoided
R=3/2 Cylinder



Isometric Embedding Diagrams

Hobill, Bernstein, and Smarr; Cox and Idaszak--NCSA Video (1988)

Similarities and Differences Between the One and Two Black Hole Problems

Property	One Black Hole	Two Black Holes
Initial Data	$K_{ij} = 0 \qquad \gamma_{ij} = \psi^4 \delta_{ij}$	$K_{ij} = 0 \qquad \gamma_{ij} = \psi^4 \delta_{ij}$
Constraint Equation	$\Delta\psi = 0 \qquad \psi = 1 + \frac{M}{2r}$	$\Delta\psi = 0 \quad \psi = 1 + \Sigma \operatorname{csch}(n\mu_o)[\frac{1}{+r_n} + \frac{1}{-r_n}]$ $\pm r_n = \{R^2 + [Z + \coth(n\mu_o)]^2\}^{\frac{1}{2}}$
Parameters	Total Mass M	M , $L/M = f(\mu_o)$ = proper distance between throats
Topology		
Lapse Function	$\alpha \to 1$ as $r \to \infty$ $\alpha = 0$ on throat → static slicing $\partial_i \alpha = 0$ on throat → dynamic slicing	$\alpha \to 1$ as $r \to \infty$ $\alpha = 0$ on throat } dynamic $\partial_i \alpha = 0$ on saddle } slicing
Spacetime Metric	$ds^2 = -\alpha^2 dt^2 + \psi^4 (Adr^2 + Br^2 d\Omega^2)$	$ds^2 = -\alpha^2 dt^2 + \psi^4 (AdZ^2 + BdR^2) + 2CdRdZ + DR^2 d\phi^2$

Smarr, Sources of Grav. Radn (1978), p.268

Roughing Out the Two Black Hole Collision Spacetime

- **Three Runs to Span the Solution Space**
  - **Run I μ_o=2.00 (Already Merged)**
  - **Run II μ_o=2.75 (Near Collision)**
  - **Run III μ_o=3.25 (Far Collision)**
- **Calibrate Using Known Solutions**
  - **Newtonian Collision**
  - **Schwarzshild Slicing**
  - **Brill Waves**
  - **Black Hole Ringing**

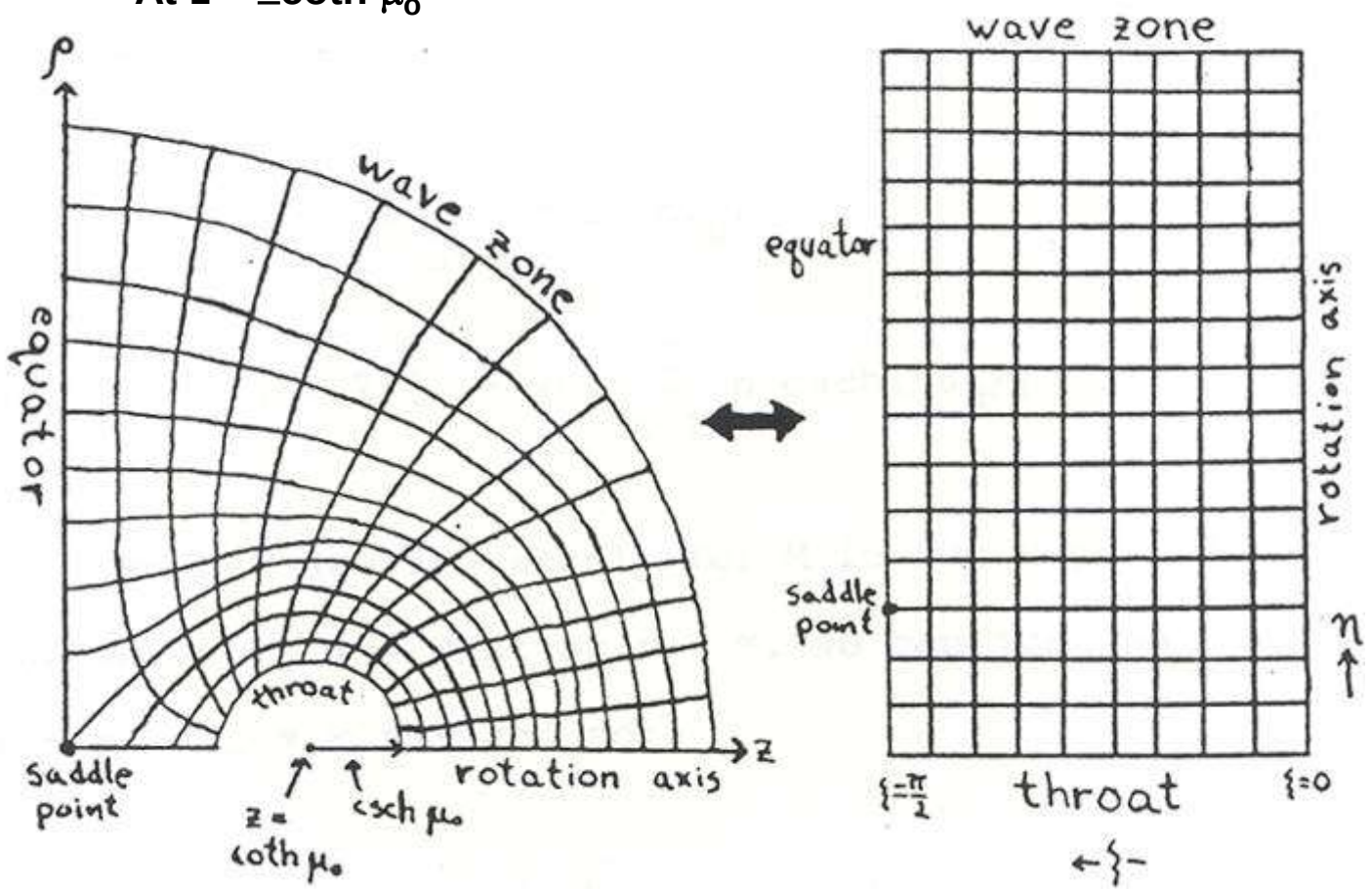
Cadez Coordinates in Terms of Cylindrical Coordinates

$$\eta + i\xi = \chi(z + i\rho) = \ell n \zeta$$

$$\eta = \ell n R = \tfrac{1}{2}\ell n (\rho^2 + z^2)$$

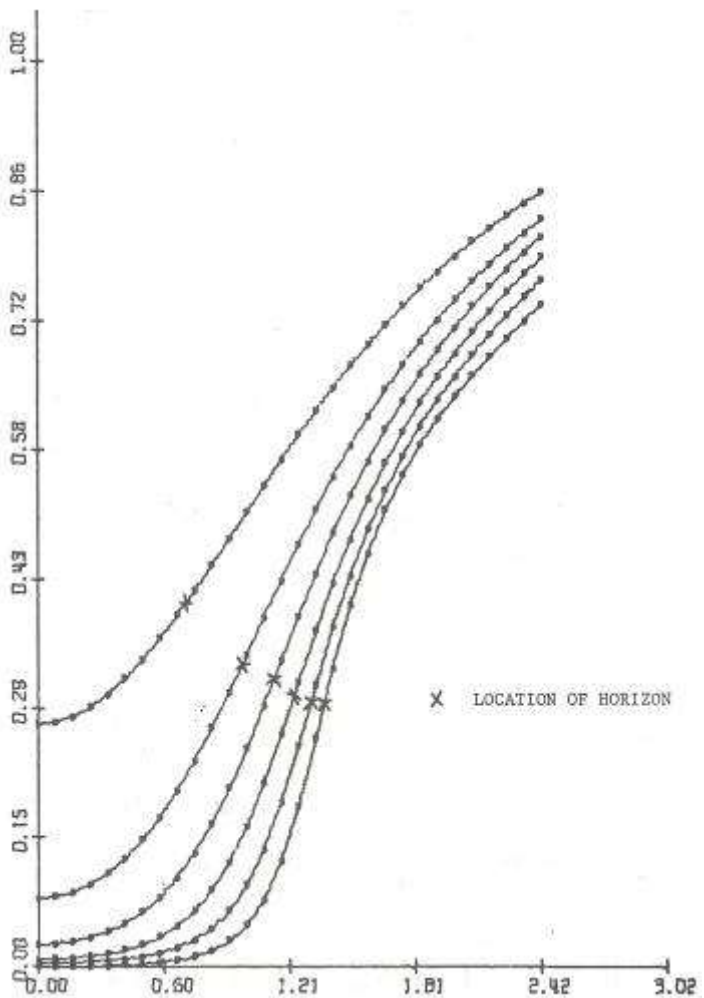
Coordinates are Field Lines and Equipotentials for Two Equal Charges At $z = \pm\coth \mu_o$

$$\xi = \theta = \tan^{-1}\left(\frac{\rho}{z}\right)$$

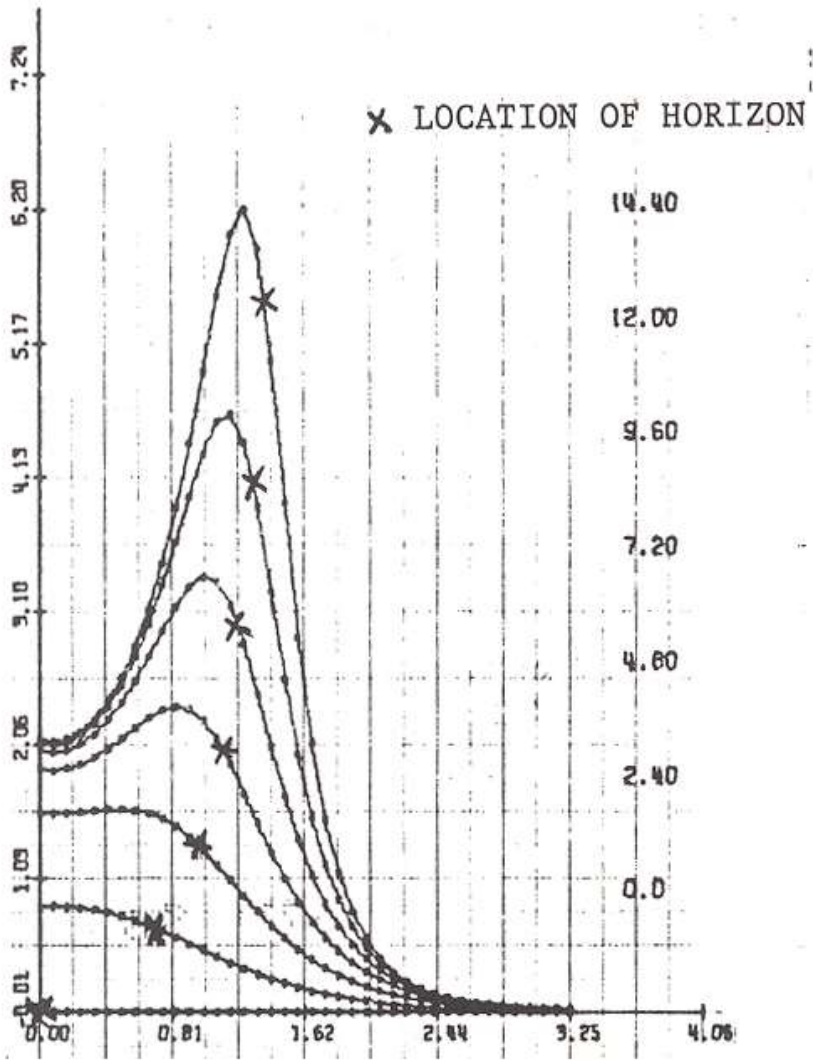


Smarr, Cadez, DeWitt, & Eppley Phys. Rev. D14, 2448 (1976)

Collapse of Lapse and Bulge in Metric for One Black Hole



Lapse Function

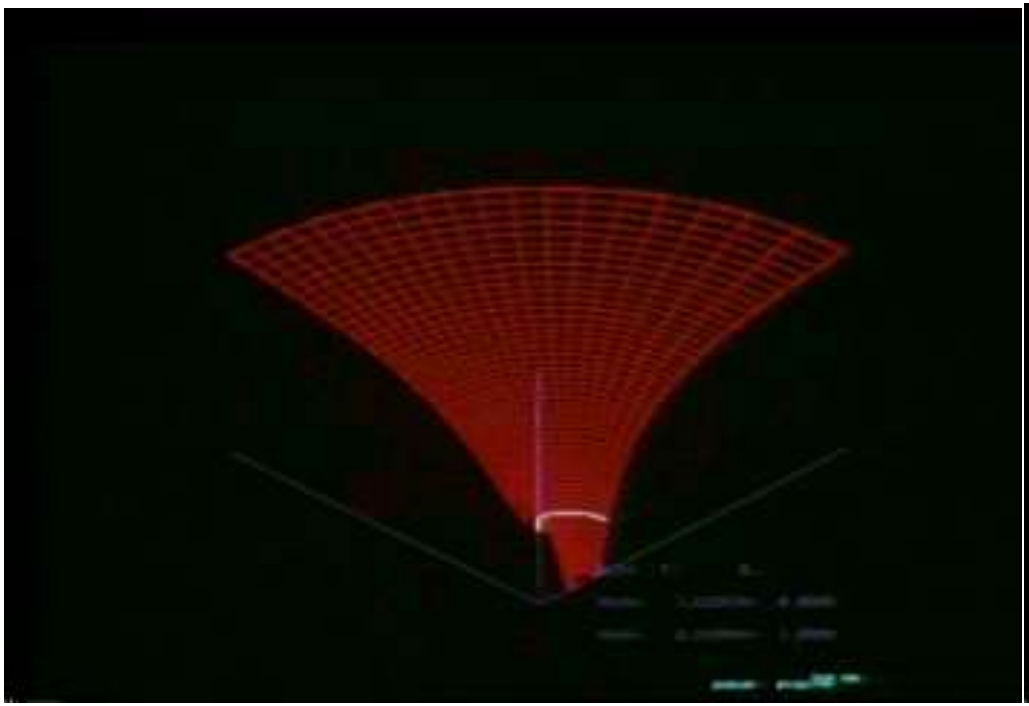


Conformal Radial Metric Function

Eppley, Ph.D. Thesis (1975), p.168-169

Collapse of the Lapse

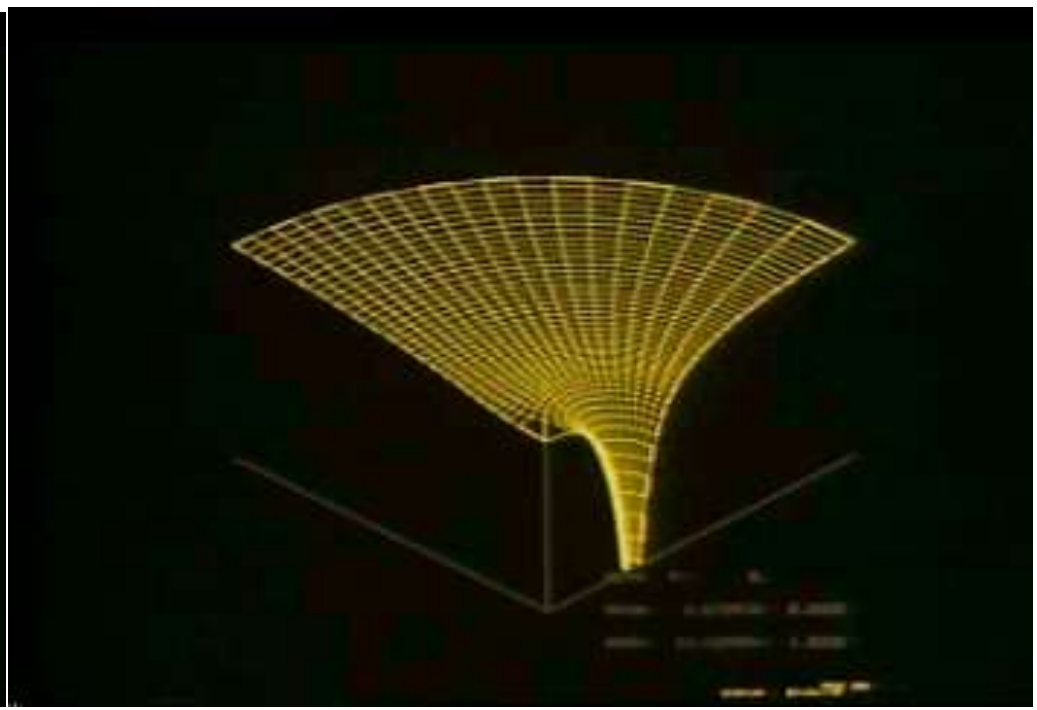
Holes Already Merged



μ_o=2.0

Outer Grid Shown is 6M

Separate Holes Collide

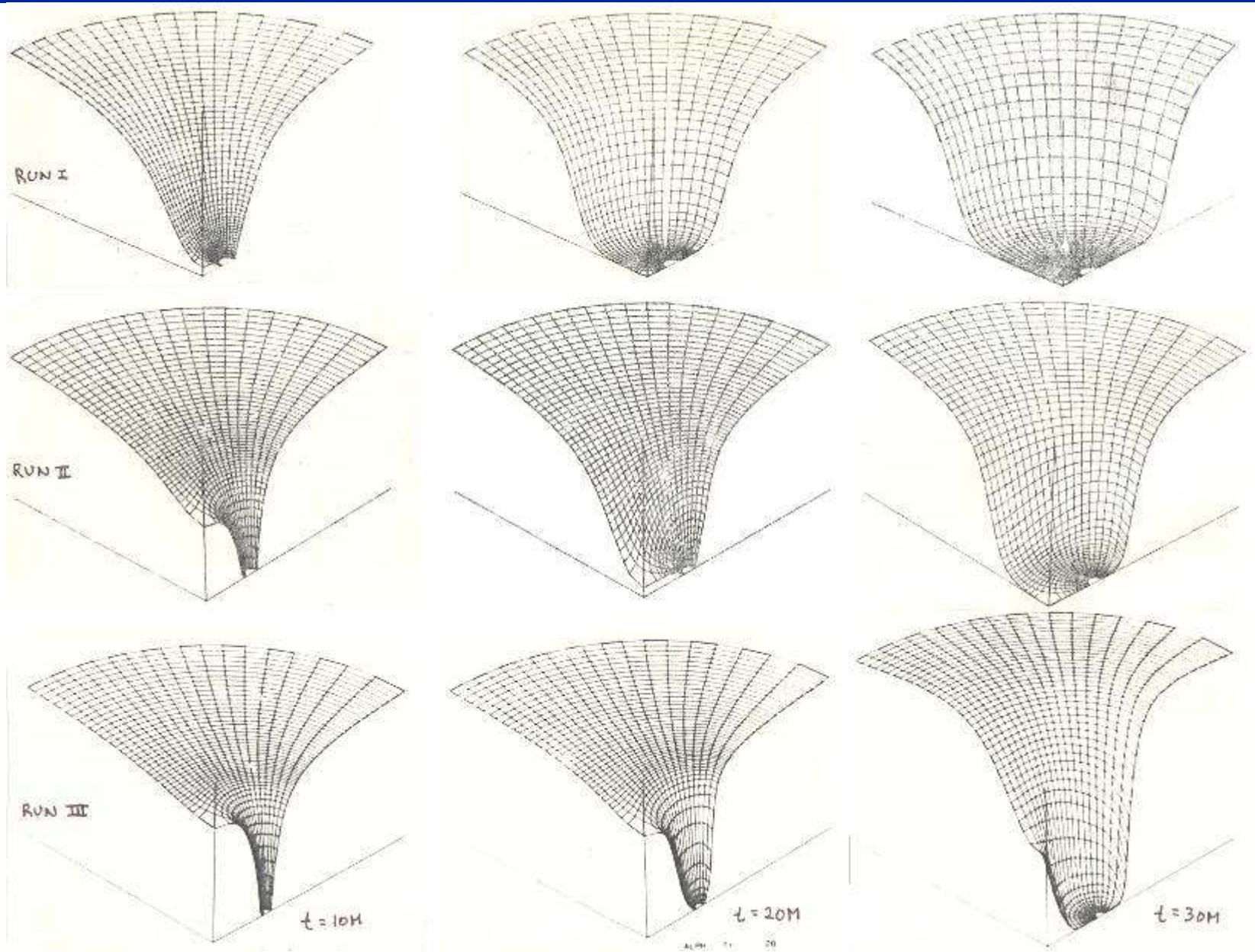


μ_o=3.75

Outer Grid Shown is 11M

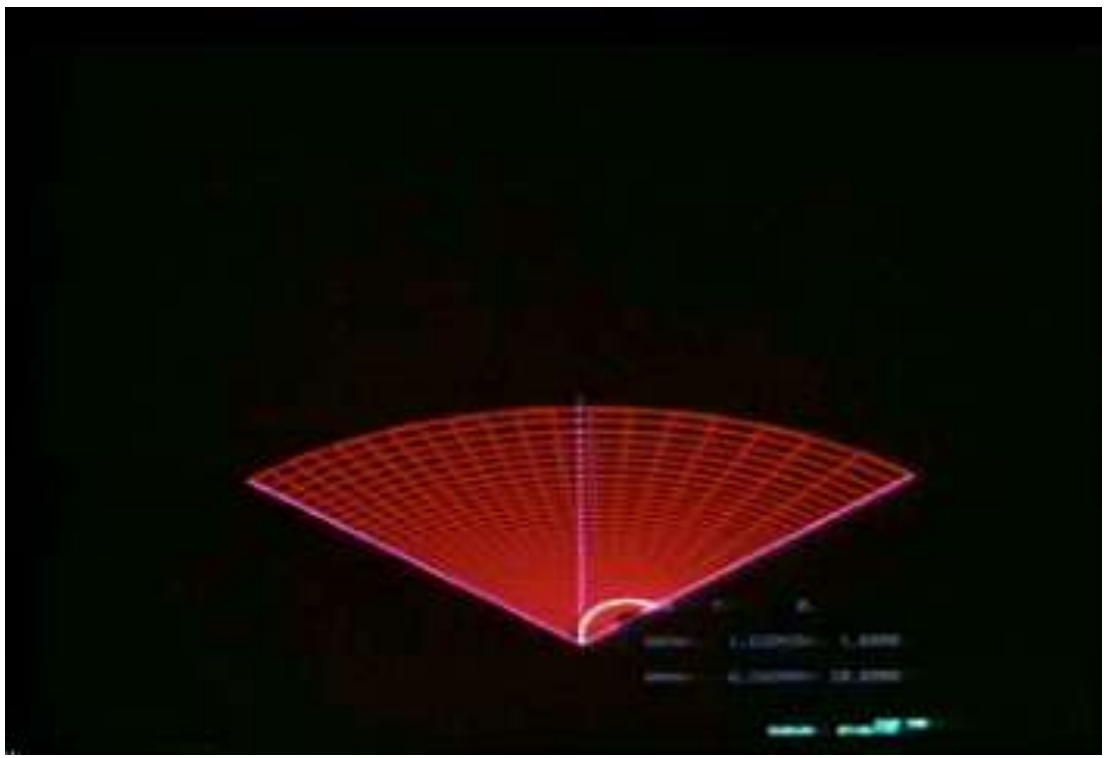
Eppley and Smarr, Research Notes (1977)

Collapse of Lapse for The Three Black Hole Collision Runs



Eppley and Smarr, Research Notes (1977)

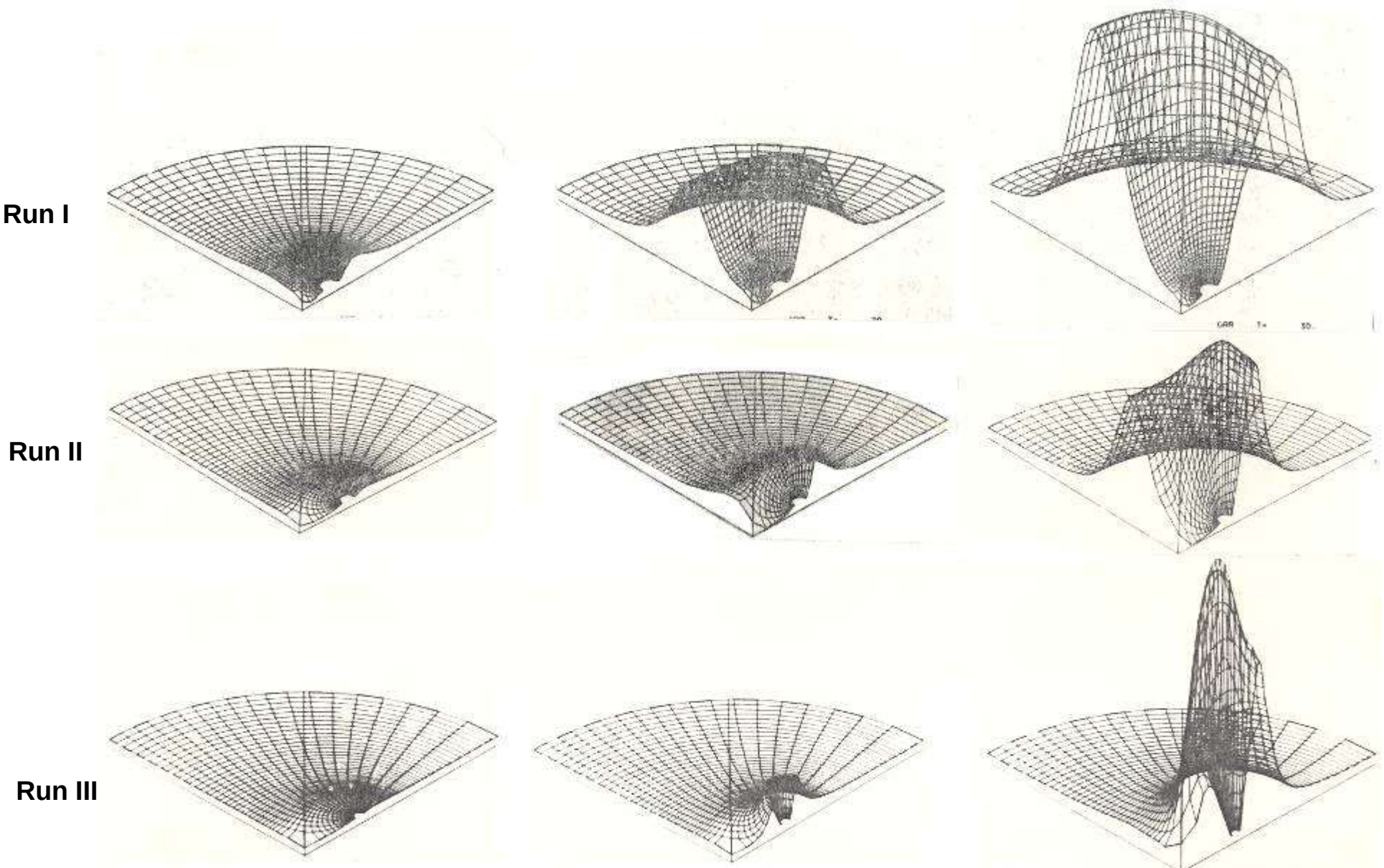
Conformal Radial Metric for μ_o=2.0 Black Hole Collision



Outer Grid Shown is 6M

Eppley and Smarr, Research Notes (1977)

Grid Sucking Induced Bulge in Radial Metric for The Three Black Hole Collision Runs



Eppley and Smarr, Research Notes (1978)

Isometric Embedding of Two Black Hole Collision 3-Space

μ_o=5.0

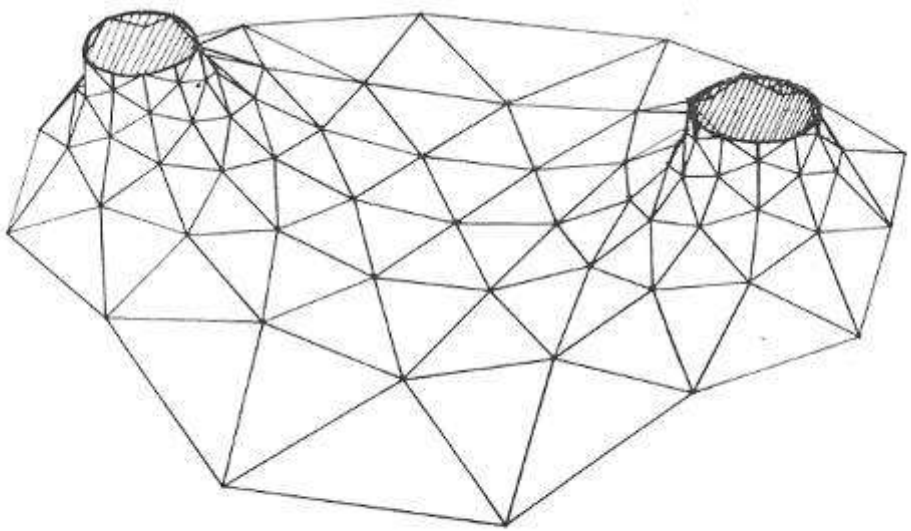
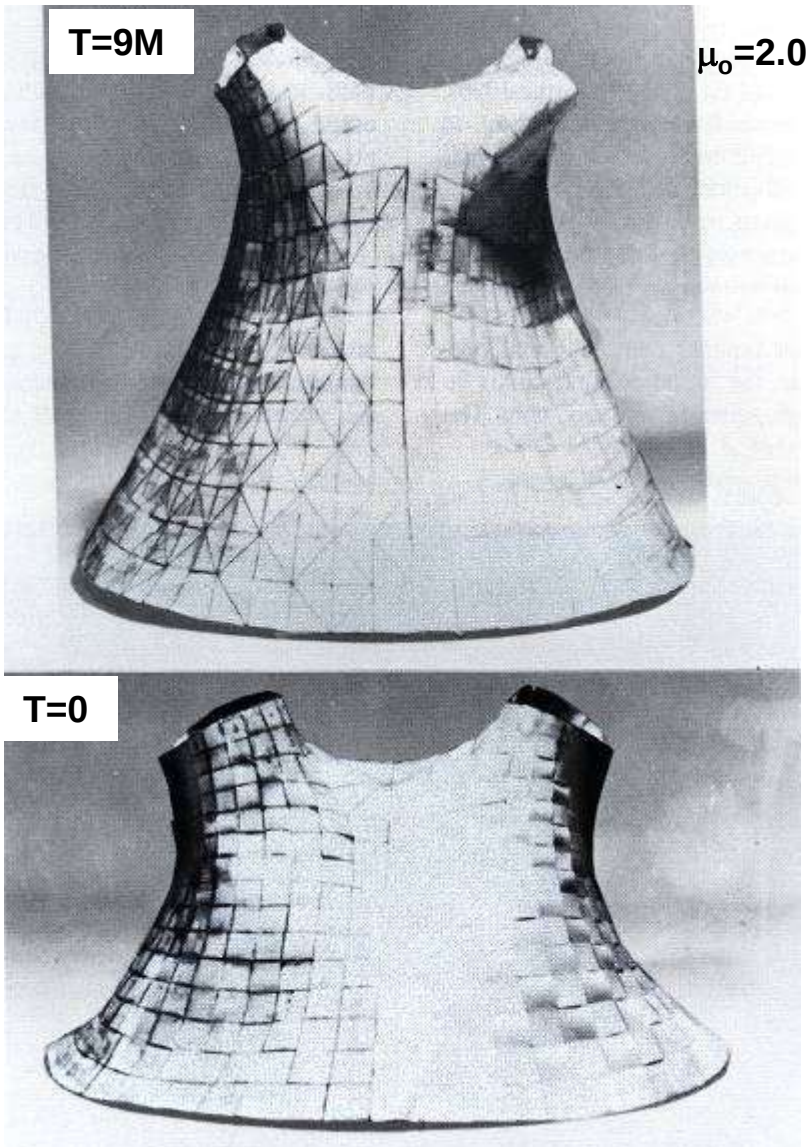


FIG. 2. The geometry of the time symmetric slice of a spacetime with two black holes. In the construction of this figure, m_1 was taken equal to m_2 and $a/m_1 = 18.8$ (see Eq. (10)).

Cadez, Ann. Physics, 91 p. 62 (1975)



Smarr, 8th Texas Symposium, p. 597 (1977)

Eppley, Ph.D. Thesis (1975), p.239

Gravitational Radiation From Colliding Black Holes

- 1959 Brill, Bondi, Weber, Wheeler, Araki
  - Time Symmetric Gravitational Waves
- 1971 Press
  - Existence of Normal Modes of Black Holes
- 1971 Davis, Ruffini, Press, Price
  - Radn. From Particle Falling Radially Into Black Hole
- 1971 Hawking
  - Area Theorem Upper Limits on Grav. Radn. From 2BHs
- 1972 Gibbons, Schutz, Cadez
  - Area Theorem Uppers Limits for Two Bound Black Holes
- 1977 Teukolsky
  - Linearized Analytic Solution for Time Symmetric Waves
- 1978 Eppley and Smarr
  - Wave Forms and Amplitudes for Different 2BH Initial Data

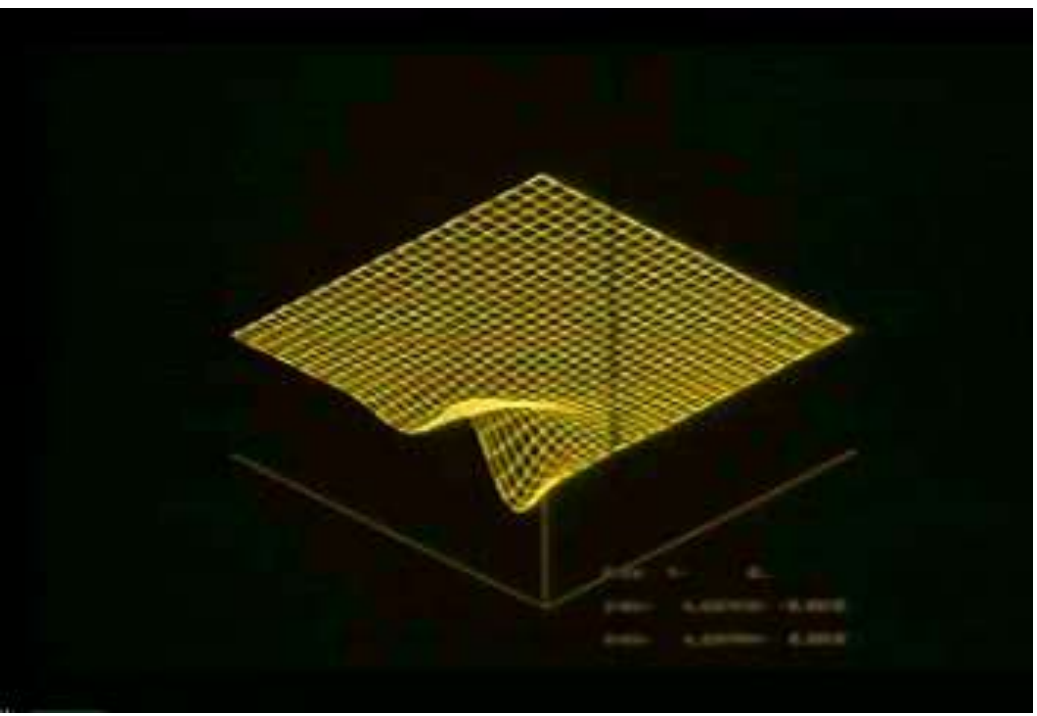
Weak Brill Waves as Code Test for Propagation of Gravitational Radiation

$$ds^2 = \psi^4[e^{Aq}(d\rho^2 + dz^2) + \rho^2 d\phi^2]$$

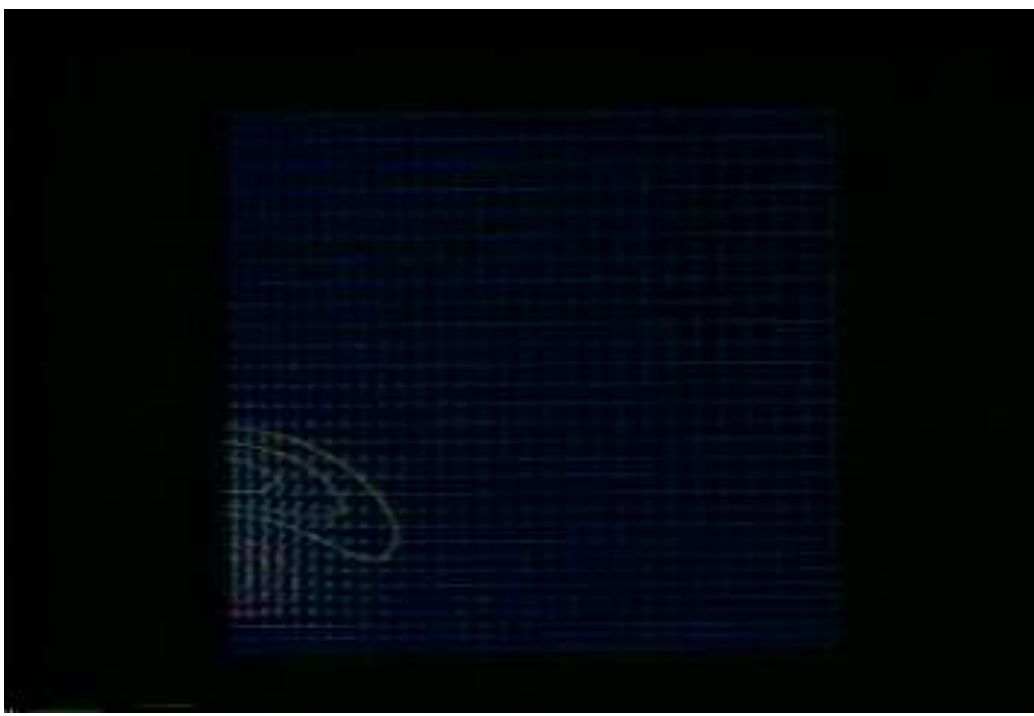
$$\Delta\psi + \Phi\psi = 0$$

$$\Phi = \tfrac{1}{4}(q_{,\rho\rho} + q_{,zz})$$

A=0.001



PSI4

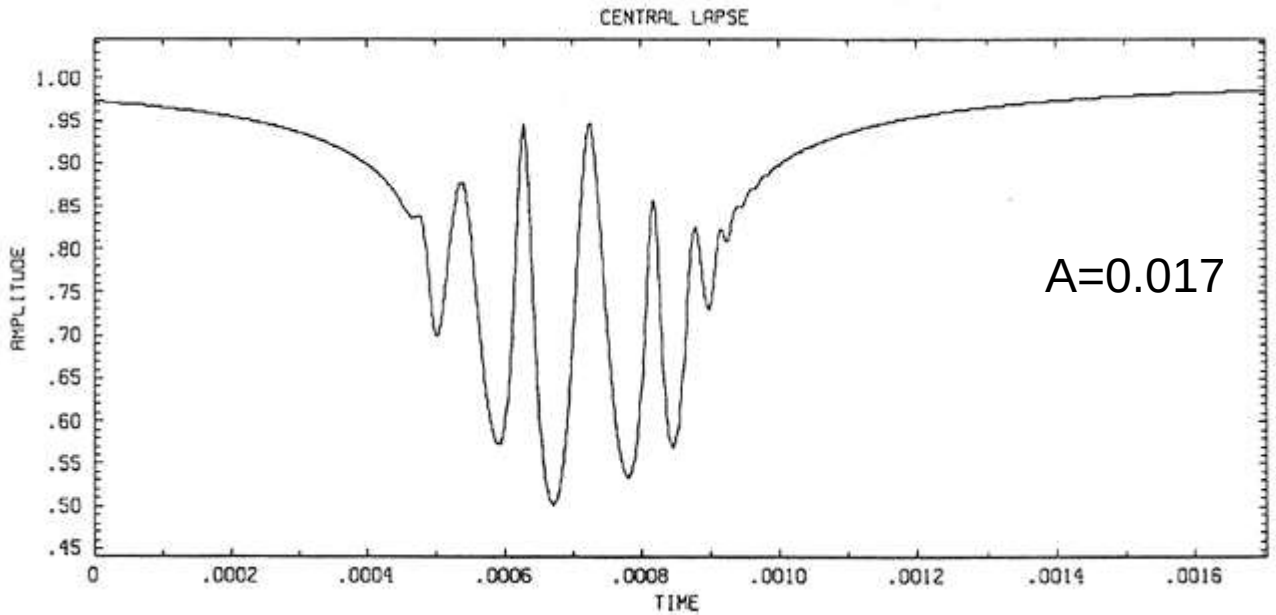


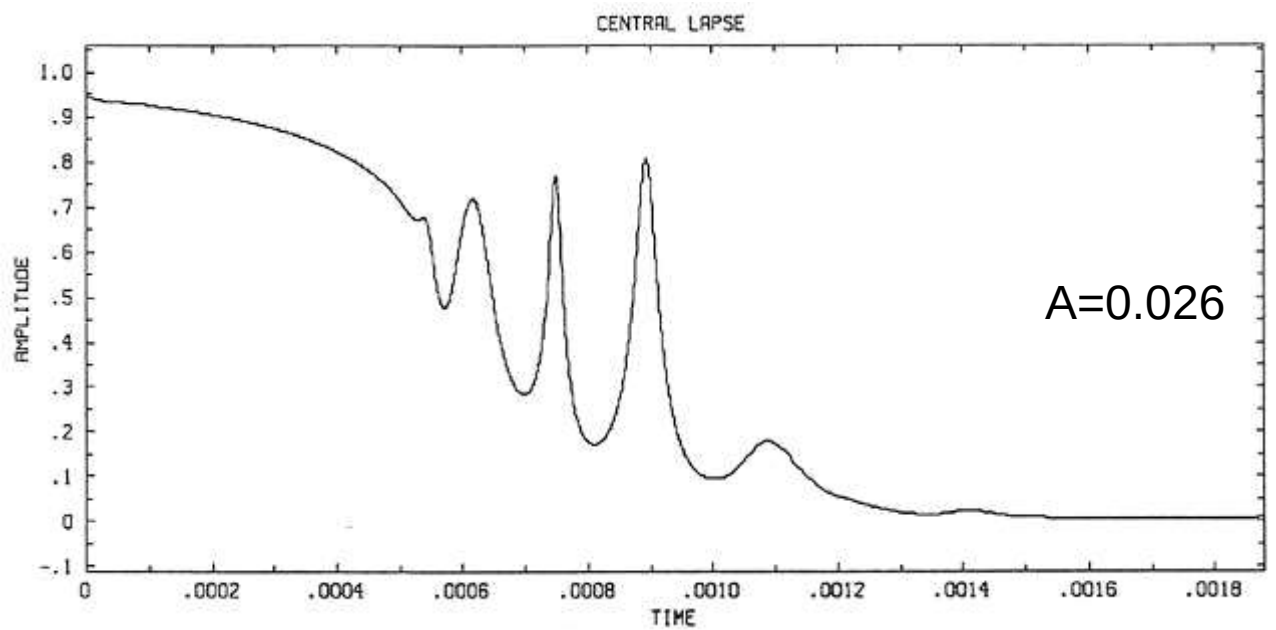
PSI4 Contours with Bel Robinson Vector

Eppley and Smarr, Research Notes (1978)

Strong Brill Waves as a Test of Gravitational Radiation Extraction from Strong Field Regions

See Also Work by Stark & Piran; Nakamura & Oohara in Mid-80s



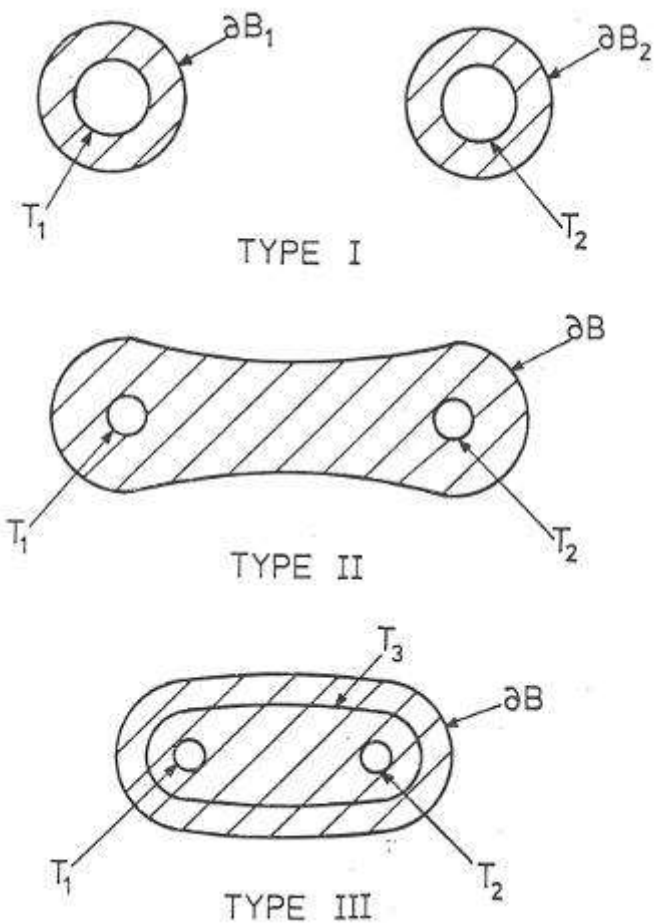


Abrahams, Ph.D. Thesis p. 98-101 (1988)

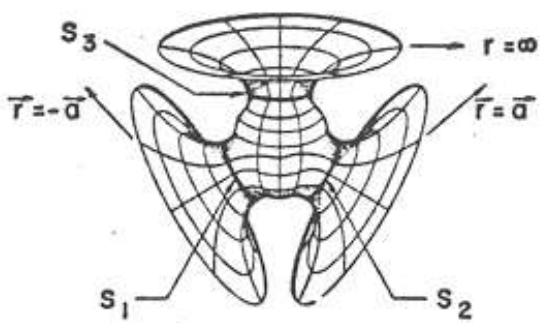
Energy Radiated for Two Black Holes, Each of Mass m

- **Hawking Area Theorem Parabolic Infall**
  - **$<0.295Mc^2$ where M is Total Mass**
- **Gibbons, Schutz, Cadez for Bound Black Holes**
  - **$<0.05Mc^2$ for $\mu_o=2.0$**
- **Newtonian Estimate for Parabolic Infall-Smarr Thesis**
  - **Two particles Fall Under Newtonian Gravity to 2M**
  - **0.005-0.02 $mc^2(m/M)$ where m is the Reduced Mass**
  - **0.0003-0.0012 Mc^2 if Masses are Equal (Range-Red Shift)**
- **DRPP Result $0.01mc^2(m/M)$**
  - **Where m is Particle Mass and M Black Hole Mass**
  - **Estimate of 0.0006 Mc^2 if m is Reduced & M is Total Mass**
- **Eppley and Smarr Computation**
  - **~0.0002 to 0.001 Mc^2 for the Three Values of μ_o**

Relationship of Event Horizon and Extreme Trapped Surfaces for 2BH Initial Data

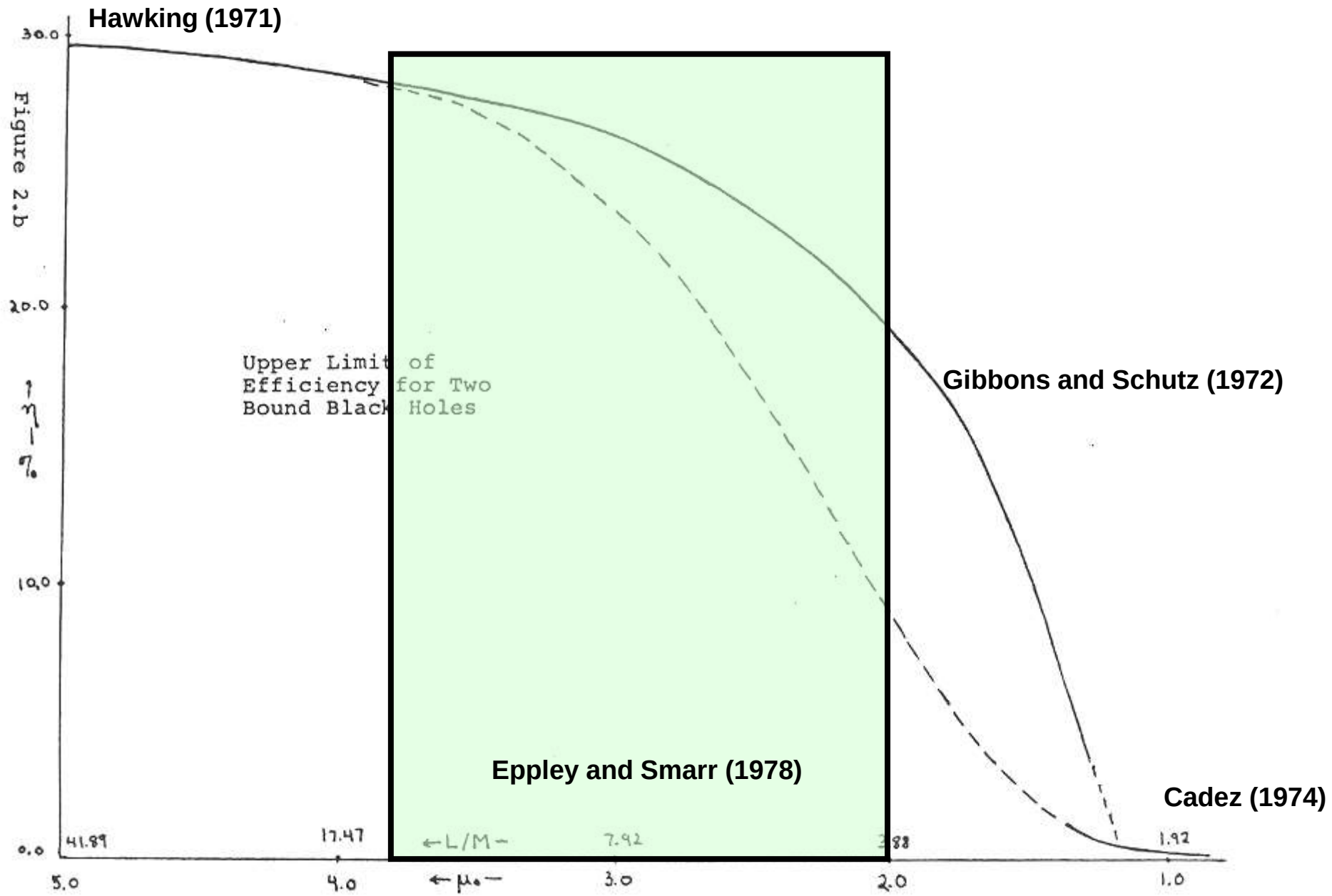


Gibbons and Schutz, MNRAS, 159, p. 41P (1972)



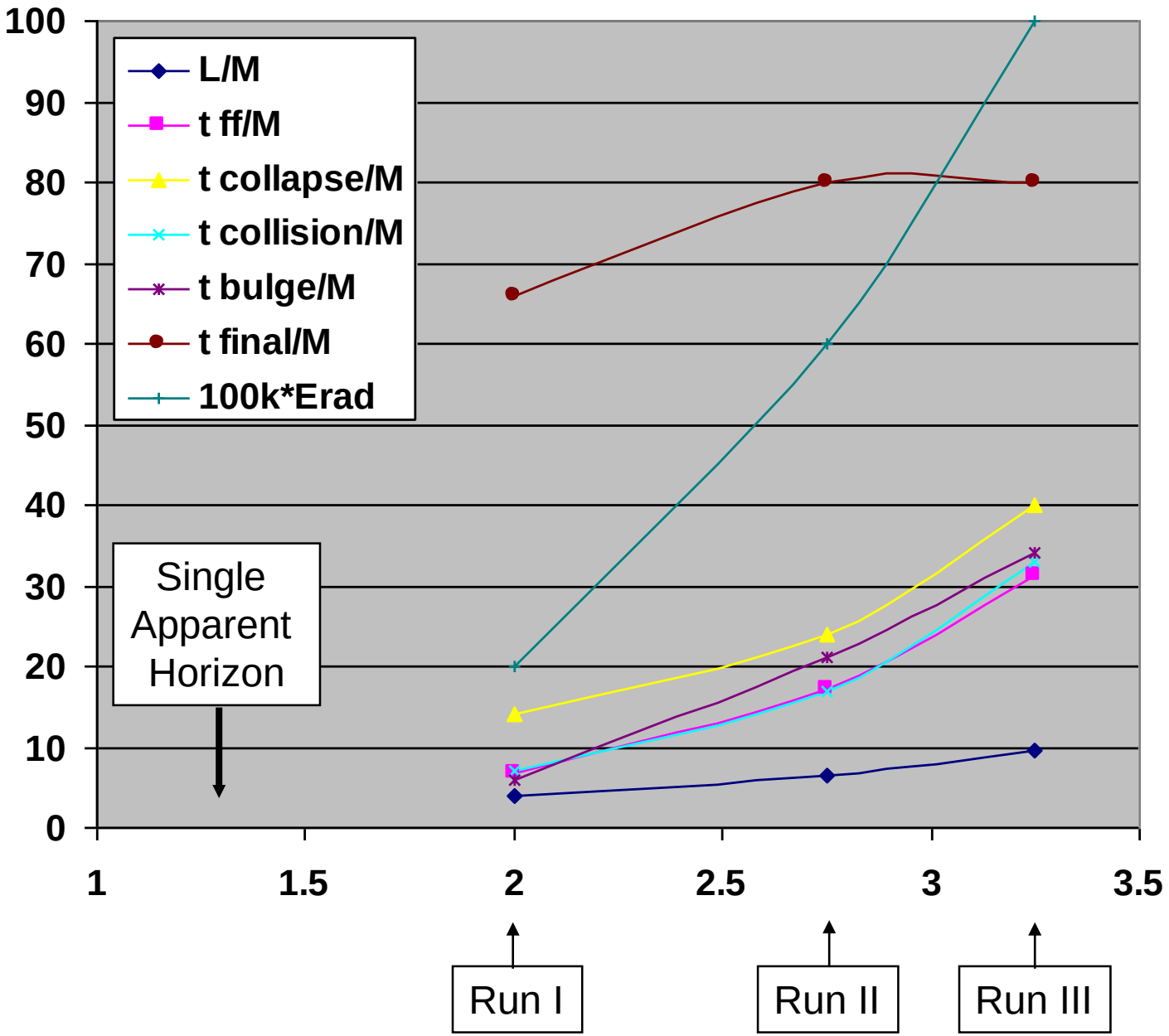
Brill and Linquist, Phys.Rev, 131, p. 472 (1963)

Hawking Area Theorem Upper Limits to Grav. Radn. Efficiency from Bound 2BH Collision



Smarr, Ph.D. Thesis (1975), p.135

Time Estimators for the Two Black Hole Collision Runs

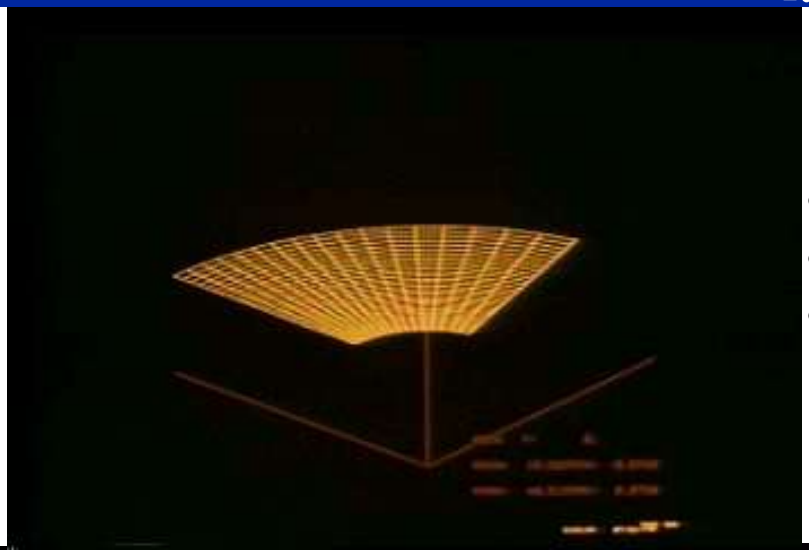


- t ff Is newtonian freefall time for L/M to 2M
- t collapse is time for lapse to drop to <0.05
- t collision is t collapse minus the 7M it takes for lapse to collapse inside single black hole
- t bulge is time for radial metric to form bulge on equator
- t final Is length of supercomputer run
- 100k*erad Is 100,000 Times the Radiated Energy

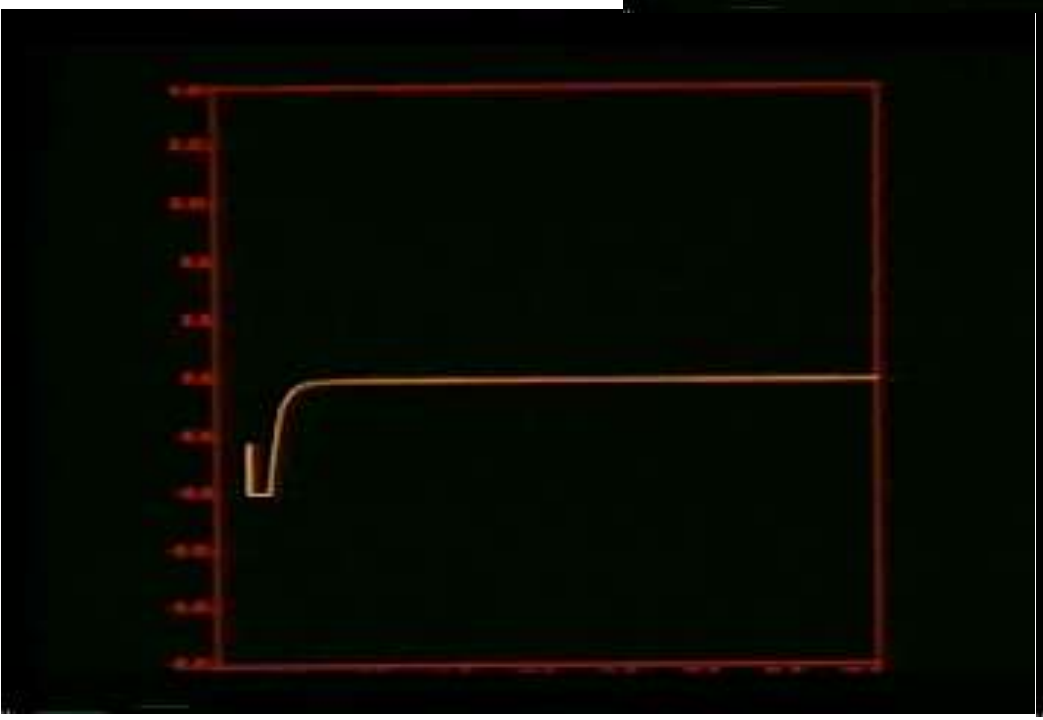
Larry Smarr, 1/18/00

Gravitational Radiation From Two Black Holes (μ_o=2.0)

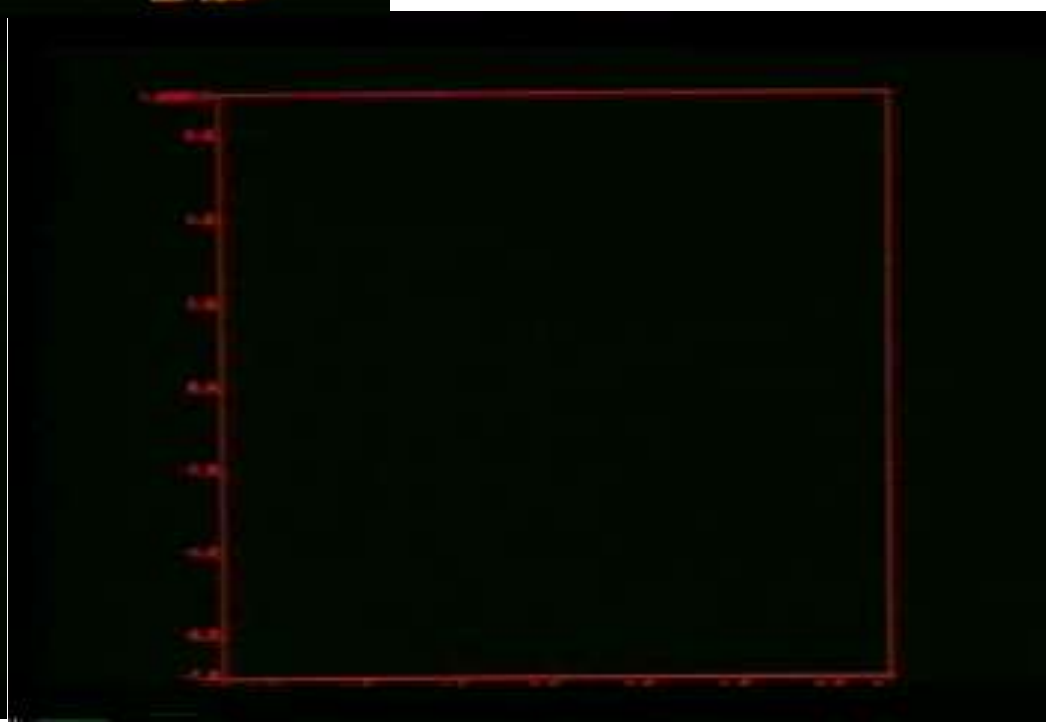
PSI4
Note Quad
Angular Pattern



- Inner Sphere Cutout at 11M
- Radiation 2-Sphere at 25M
- Grid Outer Boundary at 40M



PSI4
Along Equator



Bel Robinson Vector
Log of Radial Component

Comparison of Two Black Hole Waveform and the DRPP Perturbation Waveform

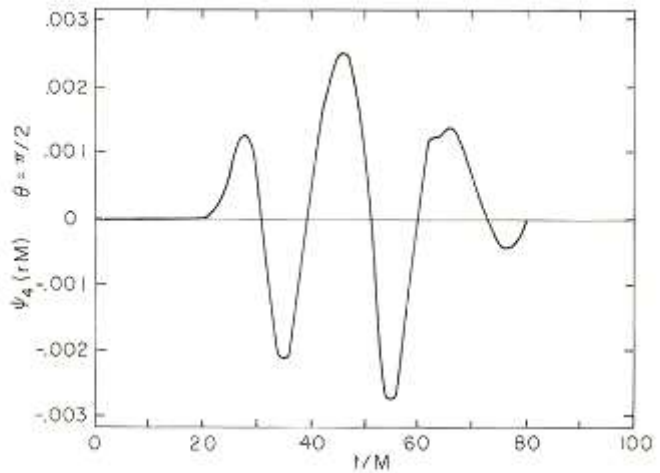


Figure 3. The curvature ψ_4.rM in the equatorial plane crossing the 2-sphere at r = 25M as a function of time. This is for the two black hole collision Run II.

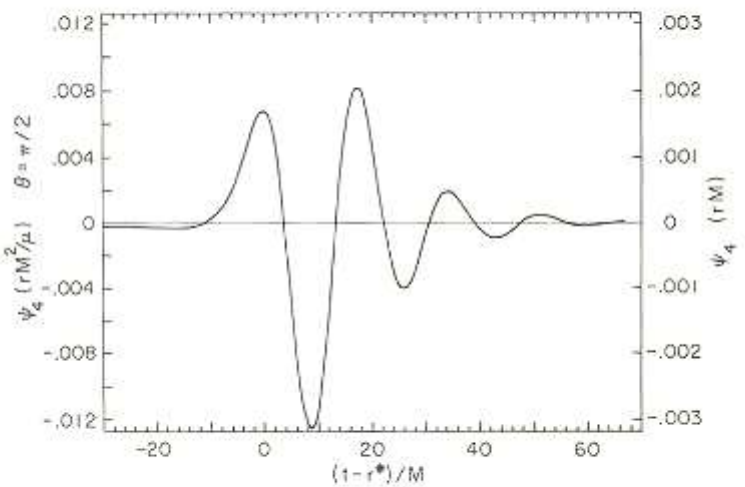


Figure 4. The same quantity as in Figure 3 except from the perturbation calculation of a particle of mass μ falling into a black hole of mass M. The abscissa is retarded time. The vertical scales are explained in the text. Only the quadrupole contribution is shown here.

Smarr, Sources of Grav. Radn (1978), p.268

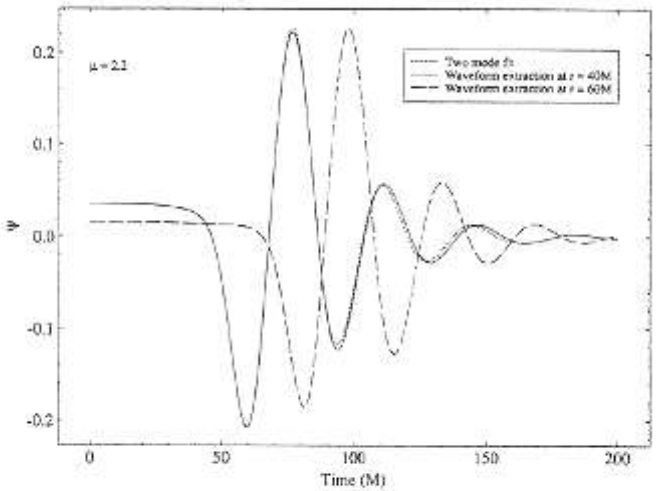


FIG. 1. $l=2$ wave forms for the case $\mu=2.2$. The solid line shows the wave form extracted at $R=40M$ and the long-dashed line shows the wave form at $R=60M$. The short-dashed line shows the quasinormal mode fit.

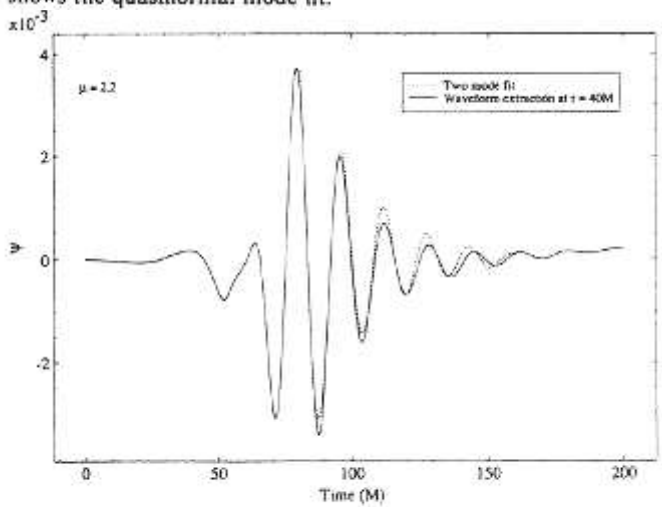
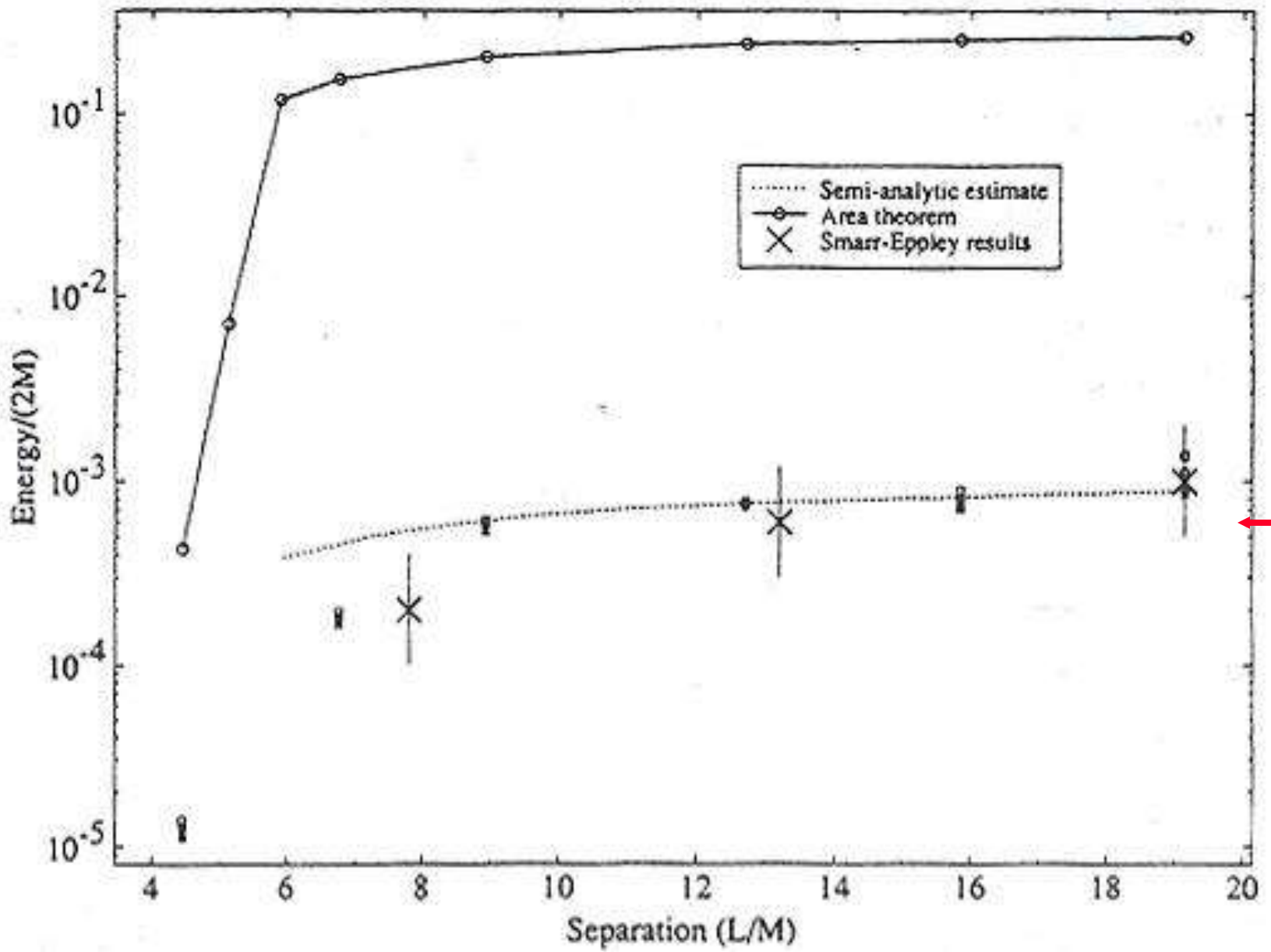


FIG. 2. $l=4$ wave forms for the case $\mu=2.2$. The solid line shows the wave form extracted at $R=40M$ and the short-dashed line shows the quasinormal mode fit.

Anninos, Hobill, Seidel, Smarr & Suen, Phys. Rev. Lett. 71, 2852 (1993)

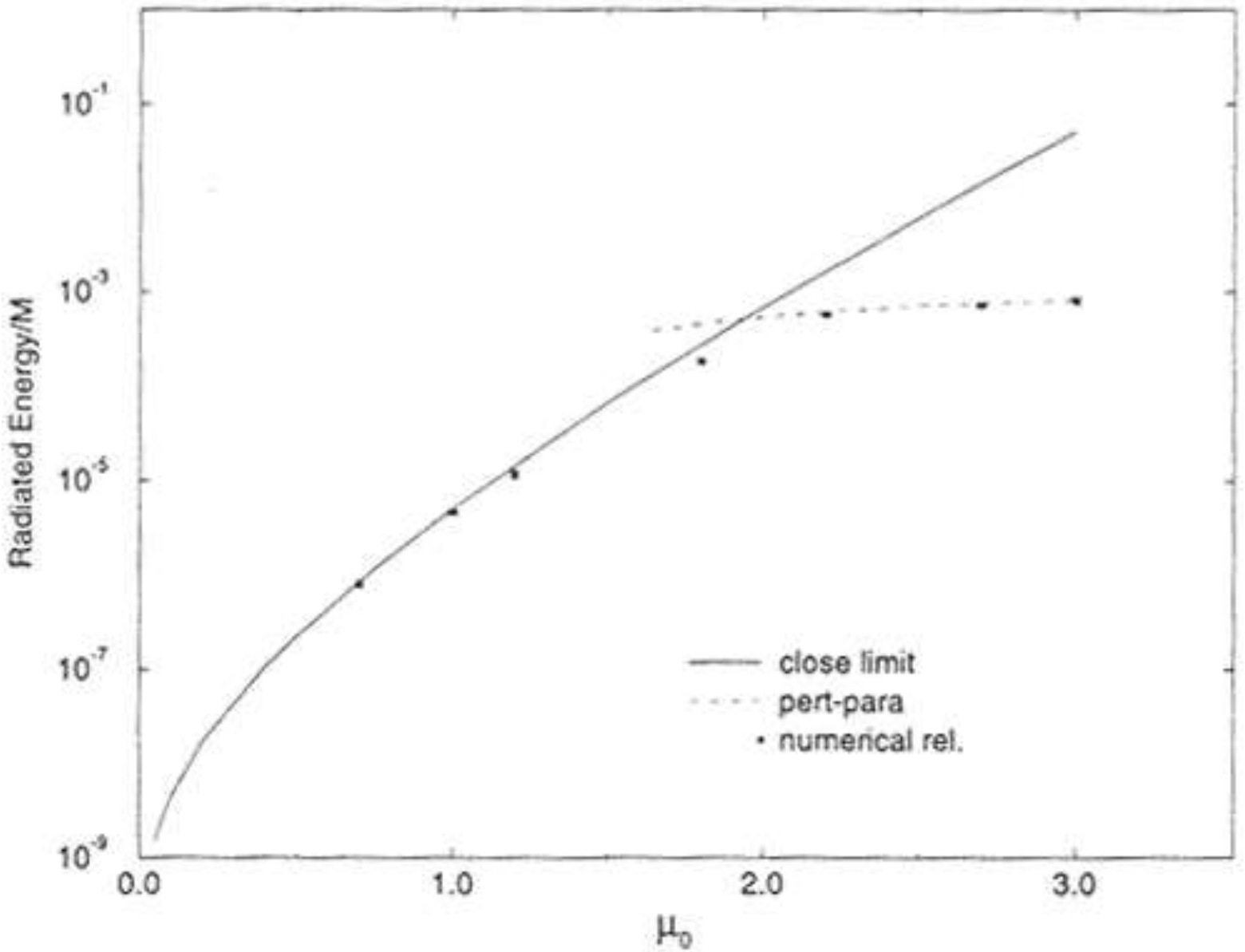
Energy Radiated From Two Black Hole Collision Compared to Area Theorem Upper Limits



DRPP

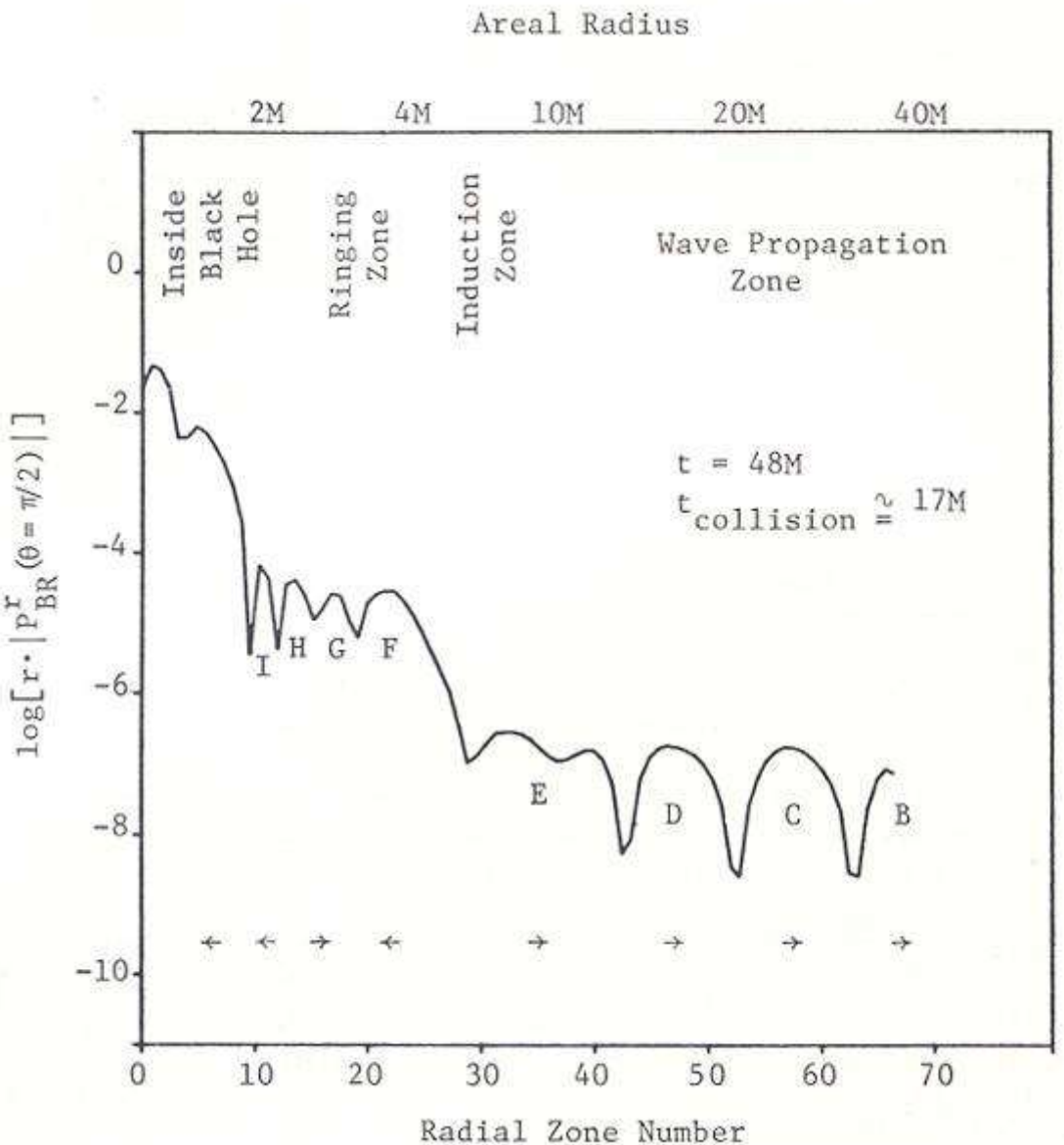
Anninos, Hobill, Seidel, Smarr, Suen, Phys. Rev. Lett., 71, p. 2854 (1993)

New Perturbation Techniques Provide Good Answers



Anninos, Price, Pullin, Seidel, Suen, Phys. Rev., D52, p. 4476 (1995)

How Well Can One Understand the Gravitational Radiation Generation Process?



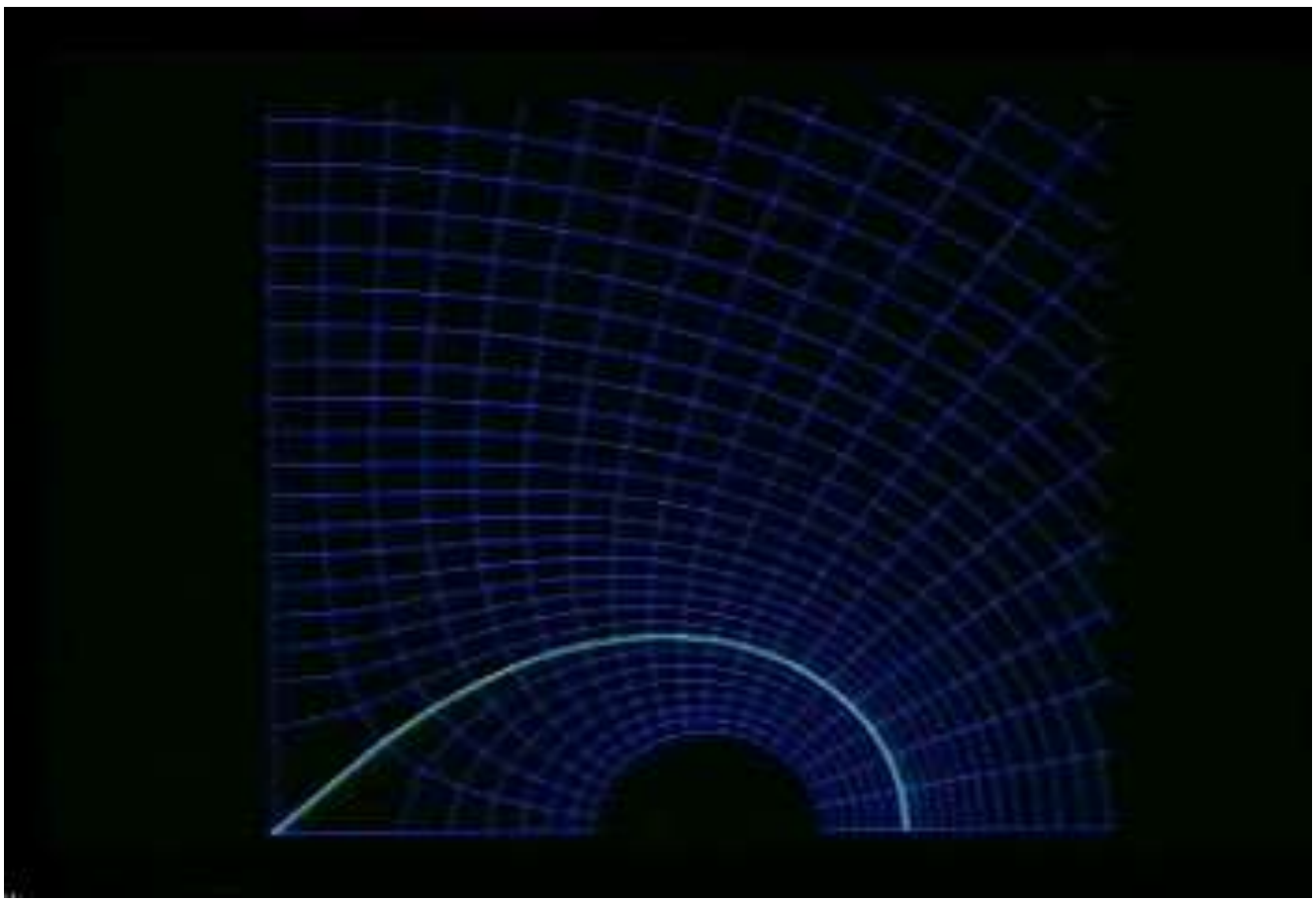
Smarr, Sources of Grav. Radn (1978), p.270

Near Zone / Far Zone
Generation of Gravitational Waves

Equator

Logarithm of Radial Bel Robinson Vector

Quadrupolar Waves



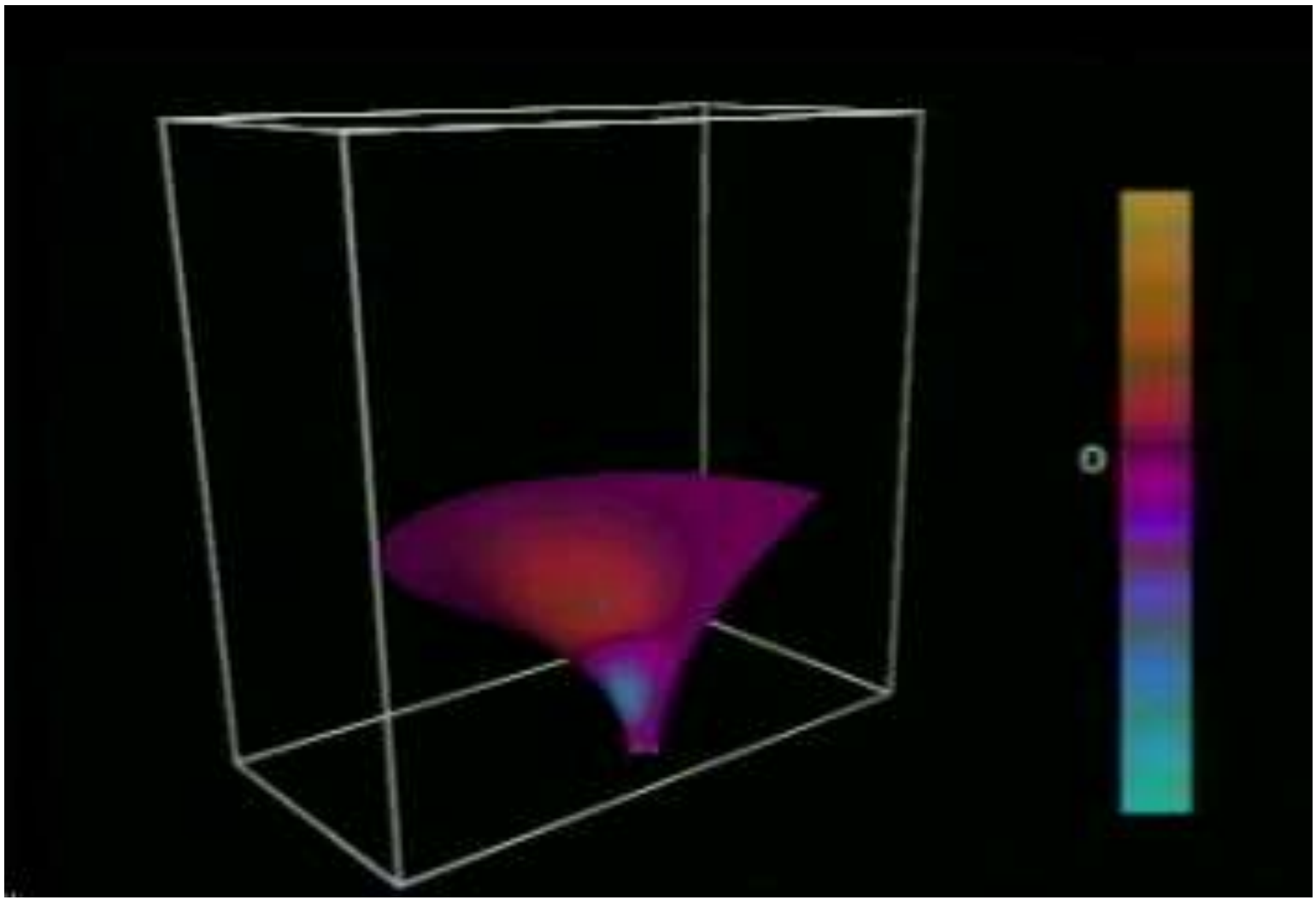
Grid Sucking

Symmetry Axis

Axis Instability

Edge of Grid is 40M

Evolution of Brill Wave / Black Hole Initial Data- Isometric Embedding Diagram



Bernstein, Hobill, & Smarr, NCSA Video (1989)

It Always Seems so Close…

For black hole [collisions] numerical relativity is likely to give us, within the next five years, a detailed and highly reliable picture of the final coalescence and the wave forms it produces, including the dependence on the hole's masses and angular momenta. Comparison of the predicted wave forms and the observed ones will constitute the strongest test ever of general relativity. (The wave forms for the astrophysically unlikely cases of head-on collisions of two identical non-rotating black holes or neutron stars have already been evaluated by numerical relativity).

--Kip Thorne, in 300 Years of Gravitation, ed. Hawking and Israel p. 379 (1987)

Bernstein, Hobill, & Smarr, NCSA Video (1989)

Problems Begun by 1985
But Unfinished…

- Strong Brill Wave Collapse and Radiation
- Axisymmetric Collision
  - Unequal Mass Head-on
  - Non-Conformally Flat, Non-Time Symmetric
  - Rotating, Boosted, and Charged Head-on
    - Cosmic Screw
  - Brill Wave and Black Hole Evolution
  - Approximation Techniques
- Non-Axisymmetric Collision
  - Rotating Holes
  - Grazing Holes
  - Orbiting Holes

Techniques Begun by 1985
But Unfinished…

- Equations of Motion
- Non-Maximal Slicings, Non-Zero Shift Vectors
- Multi-level Adaptive Grids
- Event and Apparent Horizon Location
- Full Gravitational Wave Characterization
- Better Visualization Techniques
- Community Relativity Codes

His Theories Will Keep Up Busy for the Next Century as Well!

