Fluid Dynamical Mechanisms for Knots in Astrophysical Jets

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ABSTRACT

Knotty jets recently discovered in a number of both young and evolved stellar systems show remarkable similarities in appearance to knotty extragalactic jets of vastly larger dimensions. In every case, knot emission in stellar jets is found to be shock-excited, which is strongly suspected to be the case in extragalactic jets as well (e.g. M87). We argue that fluid-dynamical instabilities (pinching and kinking) grow to nonlinear amplitude in knotty astrophysical jets and produce large-scale shock systems of the strength and distribution necessary to yield the observed knots. The behavior of fluid instabilities in the nonlinear regime and their saturation through shock formation is demonstrated by 2-dimensional gas dynamical simulations.

I. KNOTTY JETS - A UNIVERSAL PHENOMENON

An exciting development in observational astronomy over the past five years has been the widespread discovery of jets of matter emanating from astrophysical objects as diverse as galaxies and quasars (e.g. Miley 1980; Bridle and Perley 1984), evolved binary star systems (e.g. Margon 1982; Tapia et al. 1983), and very recently, pre-Main Sequence stars (e.g. Mundt and Fried 1983). Although the jets in the

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giant elliptical galaxy M87 (cf. Fig. 1a) and quasar 3C273 have been known for decades, these objects were, until recently, considered astronomical oddities. Now it seems that these objects are merely conspicuous examples of a universal phenomenon.

A universal phenomenon would seem to require a universal mechanism. This mechanism has three component pieces: the central energy source, the jet-forming mechanism, and the jet propagation to distances large compared to the source size. In all cases the ultimate energy source is the release of gravitational potential energy by the accretion of gas onto a compact object. This object can evidently be a young star, a white dwarf, a neutron star, or a black hole. All of these are approximately of stellar mass except the black hole, which can range from $\sim 10 M_\odot$ to $10^9 M_\odot$. Rees and his collaborators have discussed conditons under which accreting source models can be scaled from stellar to galactic power levels (Rees, Begelman, and Blandford 1981; Rees 1982).

The jet may be formed by the physical environment near the central object. Blandford reviews, in a paper in this volume, the possibilities of various kinds of accretion disks in this regard. The geometry of the disk certainly chooses two directions perpendicular to its equator. However, the physics of beam acceleration and focusing in a strong gravitational field may well involve the complex physics of viscous, radiation-coupled magnetohydrodynamics. Detailed calculations of this physics is still in its infancy (see Hawley and Smarr, these proceedings).

A different scenario, the Twin-Exhaust Model proposed by Blandford and Rees (1974), requires the central accreting object simply to provide a point source of energy and hot matter, which blows a large bubble into the interstellar medium surrounding the central object. If the pressure falls off faster in one direction than another, this bubble will break out along the path of least resistance, forming a jet. Unfortunately, direct observation of the jet-forming regions in stars and galaxies is not yet possible, which leaves the question of jet formation unanswered. What we can see are the jets themselves.

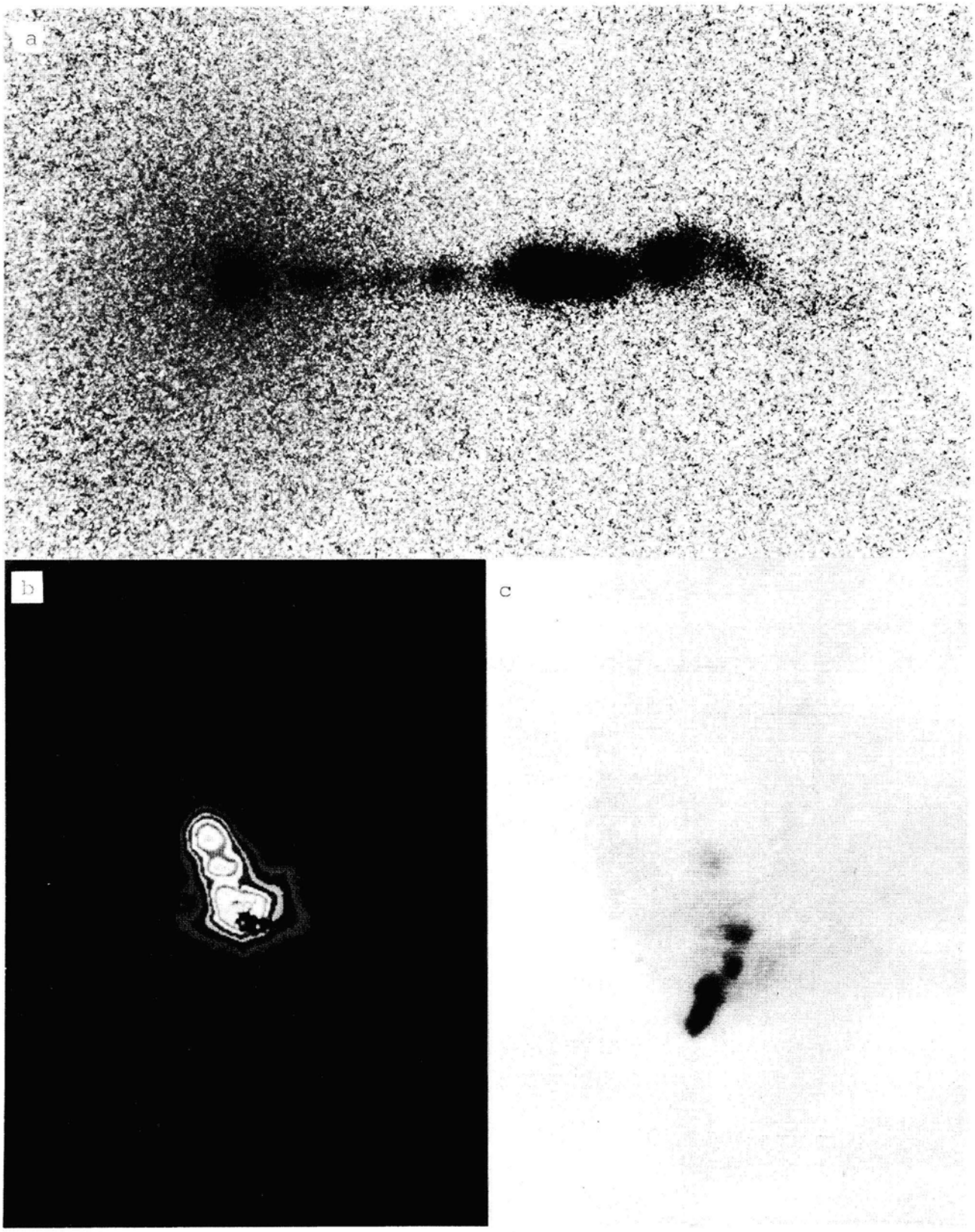


Fig. 1. - Knotty jets in a variety of astrophysical systems: a) in giant elliptical galaxy M87, U-filter, CFH prime focus (Nieto and Lelièvre, 1982): b) in symbiotic star R Aqr, Hα emission (by permission of S. Tapia); and c) in infrared source L 1551 IRS 5, r-filter (by permission of R. Mundt).

Jets are widely interpreted as continuous, highly collimated supersonic gas flows. Adopting this point of view, we therefore ask: is there observational evidence that the same underlying gas dynamics operates in jets whose lengths range from milliparsecs (stellar) to megaparsecs (extragalactic)? The existence of knots of emission with remarkably similar properties such as in the diverse objects in Figure 1 argues that the answer may be yes.

Consider the following observed properties of knots in astrophysical jets:

1. Knot emission is shock excited. In stellar jets this is readily determined due to the appearance of emission lines (Mundt and Fried 1983; Tapia et al. 1983) whose spectra are consistent with theoretical models of line formation behind radiative shockwaves (e.g. Dopita 1978). In extragalactic jets, the excitation mechanism is not unambiguously determined, although shock acceleration of the synchrotron-radiating relativistic electrons is a strong candidate (Blandford and Ostriker 1978). The recently discovered optical continuum and line emission coincident with the radio and x-ray knots in the Centaurus A jet (Brodie, Königl, and Bowyer 1983; Burns, Feigelson, and Schreier 1983) is consistent with shock excitation. But perhaps the most compelling evidence we have that shockwaves excite emission knots in extragalactic jets is the 15 GHZ VLA maps of the M87 jet by Biretta, Owen, and Hardee (1983), which show sharp intensity jumps on the upstream sides of knots A and D.

2. Knot emission in extragalactic jets is center-brightened (e.g. Fomalont 1983) indicating that the shocks responsible are large-scale (i.e. beam-filling) and not just a surface effect.

3. Knot minimum energy densities typically exceed estimates of the pressure of the confining gas (Owen, Hardee, and Bignell 1980), as one might expect downstream of a strong shock.

4. Knots often occur in fairly regularly-spaced strings, with a "wavelength" of the order of the beam diameter (Nieto and Lelièvre 1982; Biretta, Owen, and Hardee 1983), although longer spacings have also been observed.

5. Knot position and intensity frequently correlate with changes in the global structure of the jet, such as at reconfinement shoulders

(Sanders 1983; Perley, Bridle, and Willis 1984), the onset of wiggles (Nieto and Lelièvre 1982), and large-angle-deflections (e.g. wide-angle-tails, Blandford and Smarr 1983). If the beams are supersonic, such changes must have shocks associated with them.

Points 2-5 cannot be made with as much certainty for stellar jets because of the recentness of their discovery and hence the rather small sample one has to draw on. Nevertheless, this sample is growing. Mundt (private communication) has now observed a total of eight jets from young stellar objects, of which six possess knots. Although little data exists which addresses point 3, visual inspection of the CCD observations shows that the knots (which could be considered aligned Herbig-Haro objects) are consistent with the above characterization.

The above features suggest a commonality in the flow dynamics underlying the observed emission knots, namely, that knots are the observable consequences of large-scale shock systems that are excited in supersonic beams by a variety of fluid instabilities (e.g. pinching, kinking, twisting, flattening) and external influences (e.g. pressure reconfinement, bending). Because such fluid dynamical mechanisms are scale invariant, they operate in qualitatively the same way on stellar and extragalactic length scales. Dynamically important cooling, which is likely in stellar jets, will break this scale invariance by introducing a new parameter, t_{cool}/t_{flow}, which must be preserved when scaling.

The consenus among perturbation theorists who have performed linear Kelvin-Helmholtz instability analyses of supersonic beams is that their pinching modes are related to knot formation (Hardee 1979, 1982; Ferrari, Trussoni, and Zaninetti 1981a,b; Cohn 1983), although how these linear modes excite shockwaves in the nonlinear limit could not be discussed in detail. Benford (1981) has suggested that knots could be formed by the nonlinear growth of helical modes, which he argues will drive periodically-spaced shockwaves into the external medium.

The intent of this paper is to describe how the pinch and other fluid-dynamical mechanisms operate to produce shock systems of the strength and distribution necessary to yield the observed knots. The

mechanisms to be described here have all been studied assuming the flow is adiabatic (i.e., $t_{cool}/t_{flow} = \infty$). We are in the process of doing a sensitivity study to see how cooling affects the flow patterns which give rise to the knots. We will emphasize the complementary role of laboratory investigations and numerical simulations in elucidating the behavior of fluid instabilities in the nonlinear regime and their saturation through shock formation.

The remainder of this paper is organized as follows. In Section II we review briefly the numerical work done together with Jim Wilson on the Blandford-Rees Twin-Exhaust Model. This mechanism, originally envisioned for radio galaxies, is attracting renewed interest among both stellar jet (Tapia et al. 1983) and quasar jet (Narlikar and Subramanian 1983) astronomers, illustrating the utility of a scale-free theory. Section III begins our discussion of knot formation via large-scale shock systems with examples from terrestrial and laboratory experience. The properties of underexpanded supersonic jets are reviewed with an emphasis on global properties that can be related to the underlying flow variables. Sections IV and V examine the shock structures that arise in pressure-matched jets through axisymmetric and nonaxisymmetric mechanisms, respectively. Here we rely on numerical simulations for investigating flows not heretofore studied in the laboratory. An astrophysical application of these mechanisms is presented in the last section, as well as a brief assessment of the current and future role of numerical simulations in understanding the physics of cosmic jets.

II. NUMERICAL INVESTIGATION OF THE TWIN-EXHAUST MODEL

In 1974 Blandford and Rees proposed a purely fluid dynamical mechanism for producing collimated twin beams of the sort that would help explain the gross structure and energetics of the classical radio doubles which appear on either side of an elliptical galaxy, such as in Cygnus A. In the Twin-Exhaust Model, a source of constant luminosity, located in the center of a flattened gas cloud (assumed to be bound in the potential well of an elliptical galaxy), inflates a cavity with buoyant material. When the size of the inflating

cavity exceeds one scale-height of the confining gas cloud, the hot buoyant gas creates opposed de Laval nozzles through which it escapes, accelerating to supersonic velocities beyond the throats of the nozzles. The nozzles are assumed to form along the minor axis of the flattened cloud. The degree of collimation of the twin jets thus produced is determined by the run of pressure in the confining gas cloud. Left unspecified in the model was how this flow would set itself up, and whether it would be stable assuming it could be set up.

Two of us (LS and MLN), working closely with Jim Wilson, attempted a numerical investigation of the Twin-Exhaust mechanism. We turned to Jim because addressing the model's uncertainties involved dealing with the time-dependent multidimensional gas dynamics of a two-fluid system, which could be expected to develop nonlinear Rayleigh-Taylor and Kelvin-Helmholtz instabilities. Jim's experience in dealing with such complexities was, in part, embodied in a computer code that one of us (MLN) had written under his supervision. Details and findings of the numerical calculations are given in Norman et al. (1981), and analyzed in Smith et al. (1981, 1983). In the following summary, we distinguish between nozzle formation and jet collimation, which are two separate aspects of the model.

The essential results concerning nozzle formation are given in Figure 2. We verified that, given a flattened confining gas cloud with a pressure distribution near the source which falls off slower than r^{-2}, de Laval nozzles do form and can persist for a restricted range of source luminosities. If the source luminosity is too low, a thin nozzle is produced which is quickly broken into bubbles by the Kelvin-Helmholtz instability (cf. Fig. 2a). Conversely, a too high source luminosity produces such a large central cavity that a global Rayleigh-Taylor instability removes source gas in the form of large clouds (cf. Fig. 2c). The intermediate range of luminosity yields quasi-steady cavity/nozzle structures that are in qualitative agreement with the Blandford-Rees picture (cf. Fig. 2b).

The flow beyond the throat of the nozzle is either highly collimated or not, depending on the pressure-distance relation in the confining cloud. In order to fit the confining cloud on our grid,

we assumed an exponential pressure distribution. This caused the supersonic part of the flow to be poorly collimated. A power-law dependence $p \sim r^{-\beta}$ with exponent $\beta \leq 2$ would have better collimating properties.

Besides showing that jets can be formed in at least one manner, our calculations may impact on the question of the origin of knots in jets. Before the shock mechanism described in subsequent sections was proposed (Norman et al. 1982), there were two other theories of how shocks can form in astrophysical supersonic jets. The first, described by Rees (1978), relies on variations in jet luminosity or velocity in the jet forming region. He envisioned these variations leading to a series of shocks and rarefactions in the beam. The second, given by Blandford and Königl (1979), assumes that interstellar clouds could be entrained into the beam, forming bow shocks behind the clouds.

Our calculations reveal new sources for these variations or clouds. The bubble regime is probably not important in this regard because of the low velocities and energy involved. In the jet regime, however, we observe the entrainment of blobs broken off by Kelvin-Helmholtz instabilities from the nozzle wall into the flow. The cloud regime is also promising. Rayleigh-Taylor instability of the central cavity makes the flow intermittent, and also introduces large chunks of dense cloud gas into the flow. One expects this global instability to be quasi-periodic on the dynamical timescale of the confining gas cloud. This may give both the flow modulations envisioned by Rees (1978) and the obstacles required by Blandford and Königl (1979). Shock-heating of the confining cloud by the violent cavity instability may regulate the system so as to keep it near the jet regime, thus allowing quasi-continuous flow to take place.

The Twin-Exhaust Model may be the primary collimation mechanism for jets in some radio galaxies and quasars (Smith et al. 1983). Also, since nozzles may form on any scale, they may recollimate a flow that was collimated by some other mechanism on smaller length scales, but subsequently decollimated by its passage through the intervening medium. Königl (1982) has applied the Twin-Exhaust Model to bipolar flows, where the energy source is a young star with a

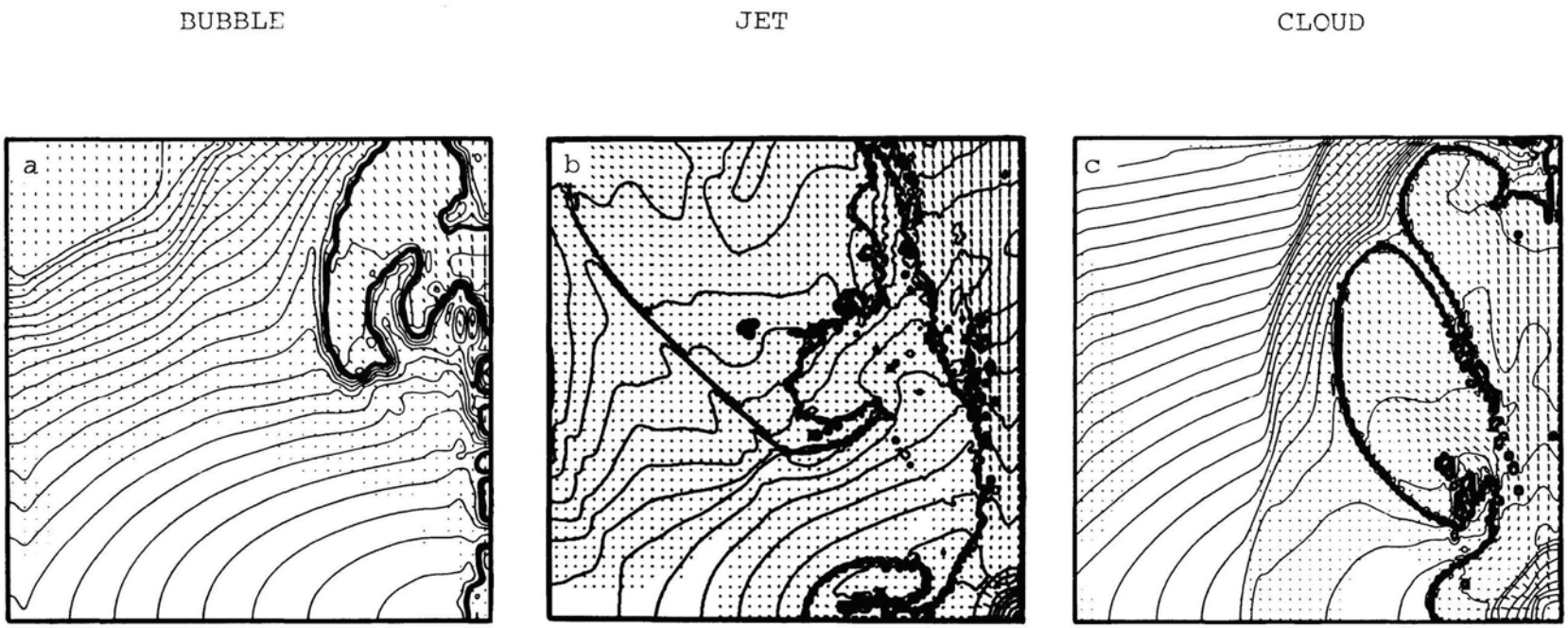


Fig. 2. - The three regimes of bipolar flow in the context of the Twin-Exhaust Model, as determined by hydrodynamical simulations. Density contours and velocity vectors are shown for one quadrant of the system. Axis of symmetry is vertical, equator is horizontal. Vectors are drawn at every 2nd zone.

strong stellar wind, and the confining agent is a dense molecular cloud. Tapia et al. (1983) suggest that the nozzle mechanism may be responsible for the opposed jets in R Aqr. Narlikar and Subramanian (1983) have proposed a "Single Exhaust Model", which they adapted from the Blandford and Rees model, to explain quasars with one-sided jets. Begelman and Rees (1983) have extended the basic model to a generic situation of an energy source swamped by outside gas, which feeds the central source by accretion. Their so-called Cauldron Model may be applicable to any compact object, regardless of scale, which produces jets fueled by a super-Eddington level of accretion. These examples illustrate the utility of purely fluid dynamical mechanisms for constructing models of related phenomena in a variety of astrophysical settings and on vastly different scales.

III. KNOTS IN TERRESTRIAL JETS

Nearly a century ago, Mach (Ernst) and Salcher (1889) noticed that the supersonic air jet they were using in their investigations of flow past projectiles exhibited a "customary appearance so remarkable that a detailed study of the jet itself seemed worthwhile". To this end, Ernst Mach's son Ludwig began a series of careful experiments that resulted in photographs such as the one reproduced in Figure 3a (Mach 1897). He observed that pressurized air escaping into the atmosphere through an orifice produces a jet which, when properly illuminated, exhibits "uniformly distributed, nearly equidistant luminous knots, whose spacing decreases with the further decrease of the reservoir pressure." When the jet was observed with Schlieren techniques, wherein density gradients in the flow modulate the intensity of the illuminating light, the knots were seen to be embedded in a network of crisscrossing lines (cf. Fig. 3b), which Mach determined were stationary compressive waves (shockwaves).

This pattern of shockwaves is characteristic of supersonic jets emerging from nozzles out of pressure equilibrium with the ambient gas (e.g. Adamson and Nicholls 1959), and is the object of study among experimental aerodynamicists and rocket designers. The shockwaves are responsible for the "beaded" appearance of the plumes of

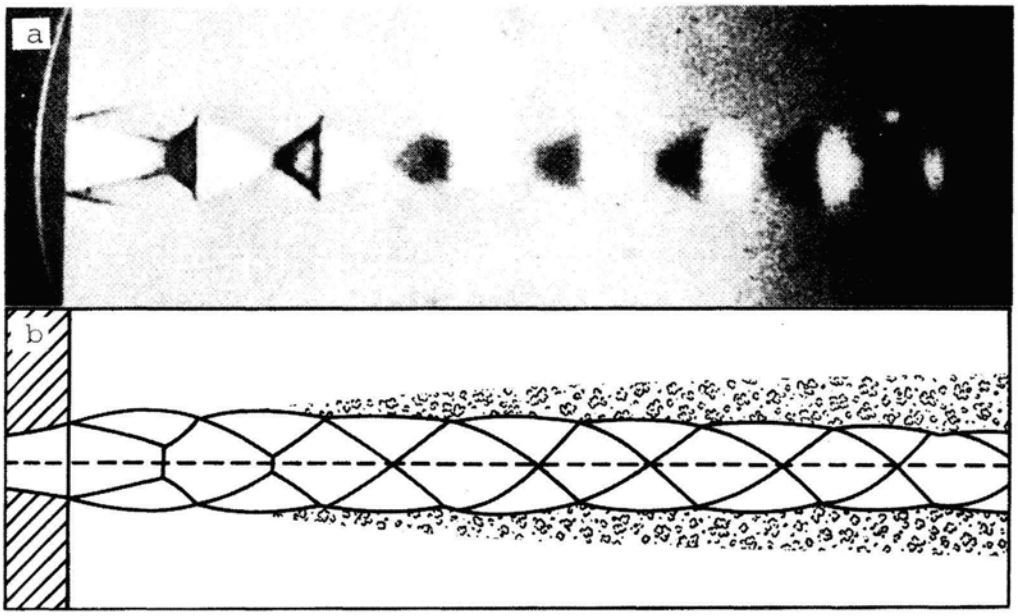


Fig. 3. - Characteristic oscillation of a supersonic airjet exiting a nozzle with greater-than-ambient pressure. a) Schlieren photograph by L. Mach (1897) showing periodic density enhancements (knots). b) Flow schematic showing system of crisscrossing shockwaves and turbulent boundary layer.



Fig. 4. - Optical knots in the supersonic exhaust plume of the rocket plane Bell X-1 (Anderson 1981).

rocket engines, such as shown in Figure 4. Indeed, Wormhoudt and Yousefian (1982) have shown that patches of luminous emission in the exhaust plumes of the Space Shuttle main engine are excited by passage of the chemicallly reactive fuel through these embedded shockwaves.

One can qualitatively understand the global shock pattern and the resulting knots as a nonlinear oscillation of a supersonic jet out of pressure equilibrium with its surroundings. A complete mathematical understanding of the phenomenon is currently lacking, because of the nonlinearities of the flow, and because of the little attention this problem has received. Modern experimental work since 1941 has been primarily engineering in nature, and has considered only the structure of the flow adjacent to the nozzle up to and including the first shock reflection (e.g. Ladenburg et al. 1949; Chang and Chow 1974). In addition, even the most comprehensive studies (e.g. Love et al. 1959) are limited to dense jets with Mach numbers in the range of 1 to 3. Here we summarize what is known about the local and global structure of the underexpanded supersonic jet. Potential applications to astrophysical jets are discussed in Section VI.

Figure 5 shows the flow structure in a supersonic jet exiting a nozzle with greater-than-ambient pressure. As the high pressure gas

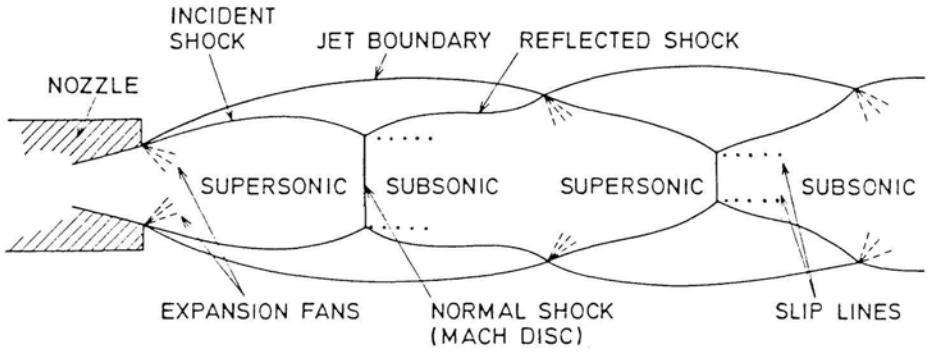


Fig. 5. - Structural details of a highly underexpanded supersonic jet, adapted from Adamson and Nicholls (1959).

leaves the confining walls of the nozzle, it undergoes a Prandtl-Meyer expansion (cf. Sec. 109, Courant and Friedrichs 1948) to decrease its pressure and match onto the atmospheric pressure. This causes the jet boundary to expand radially and the interior of the jet to rarify to lower-than-ambient pressures. This overexpansion reverses first at the jet boundary; its reconvergence drives the so-called incident shock towards the jet axis, which communicates the external conditons to the interior of the jet. Depending upon the angle of incidence, which in turn depends on the pressure ratio at the nozzle, this shock either reflects at a point on the axis (regular reflection), forming with the reflected shock a biconical structure, or, as depicted in Figure 5, reflects by forming a Mach disk. A Mach disk is a strong shock that forms normal to the flow direction and meets the incident and reflected shocks at a shock triple point. Triple shock intersections generate vorticity in the form of slip lines or surfaces. The reflected shock propagates outward, reversing the motion of radially convergent jet gas. When the reflected shock hits the jet boundary, the cycle is complete, for the gas immediately downstream of the reflected shock and Mach disk (if there is one) is again at higher-than-ambient pressures. The flow is subsonic just behind the Mach disk but accelerates to supersonic velocities in a second Prandtl-Meyer expansion zone. As is evident in Figures 3 and 4, this nonlinear oscillation can repeat itself many times downstream of the nozzle before damping.

The global structure of the jet can be quantified by the oscillation wavelength in units of the mean jet diameter and the number of oscillations before damping or disruption sets in. The first systematic investigation of the global flow pattern was made by Emden (1899), who determined that a simple relation exists between reservoir pressure P, orifice diameter D, and oscillation wavelength W, namely $W/D = 0.89\sqrt{P-1.9}$, where P is measured in atmospheres. The range of reservoir pressures Emden used in his experiments was $2 < P < 8$ atm. These experiments promted Prandtl (1904) to analyze an ideal axisymmetric supersonic beam for an oscillating mode of linear amplitude, which he found to obey the relation $W/D = 1.307\sqrt{M^2-1}$, where M is the Mach number of the beam. This relation Prandtl showed

to be consistent with Emden's empirical law for low reservoir pressures and therefore low Mach numbers (i.e., small pressure mismatches at the nozzle and weak shocks).

However, for higher reservoir pressures and Mach numbers, both Emden's relation and Prandtl's analysis break down. For example, Emden's relation predicts a W/D of 3.2 for the P=15 atm. jet in Figure 3a (Mach 1897), whereas W/D can be measured on the photograph to be ~1.7. Hartmann and Lazarus (1941) discuss this breakdown, showing that W/D saturates at roughly 2 due to the formation of strong shockwaves of the Mach-disk type for reservoir pressures P $\gtrsim$ 4.5 atm. (M $\gtrsim$ 1.6). Figure 5 shows a supersonic jet in this regime.

No detailed theory yet exists which can predict the spatial extent of the oscillations, because the damping and disrupting mechanisms are not well understood. The oscillation is damped by two dissipative processes related to the network of internal shockwaves. First, entropy is produced at shocks at the expense of directed motion. Second, because the shocks are generally nonuniform, vorticity is produced which leads to microturbulence. These two effects combine and become dominant in highly underexpanded jets (P $\gtrsim$ 20 atm.) which completely disrupt downstream of the first Mach disk (e.g. Figure 10 of Ladenburg et al. 1949). For smaller pressure ratios, the spatial extent of the oscillations is usually limited by the inward propagation of the turbulent boundary layer at the jet edge, or by the onset of large-scale nonaxisymmetric motions.

A detailed and comprehensive theory of the underexpanded supersonic jet will be difficult to achieve, for not only are the individual processes just mentioned incompletely understood, but they are also intimately coupled. Thus, the boundary layer provides an ever-broadening and time-dependent surface off of which the internal shocks reflect, whereas the rate of growth of the boundary layer is effected by the impinging shockwaves. Nevertheless, certain gross properties of the flow, such as the saturation of the oscillation wavelength at roughly two jet diameters, are fairly well determined. We discuss the astrophysical implications of this signature in Section VI. But first we turn to a summary of our recent numerical work on pressure-matched jets, where quasi-periodic internal shockwaves show up in different but related guises.

IV. KNOTS IN PRESSURE-MATCHED SUPERSONIC JETS

Many of the jets seen in radio galaxies appear to be pressure confined by the surrounding gas, and a number of these have well developed knots (e.g. Centaurus A, Burns, Feigelson, and Schreier 1983). In the last section we described how systems of shockwaves and knots are excited by the reconfinement of an underexpanded jet. Here we pose the following questions: what is the behavior of a supersonic beam of gas that exits the nozzle with the same pressure as the ambient gas? (This is termed "pressure-matched".) In particular, do internal shocks and knots arise in the absence of gross pressure mismatches between jet and ambient medium? In contrast to the usual laboratory situation, we observe astrophysical jets in the process of boring through the gaseous confining medium. Therefore, in addition to the jet's time-average properties, we are also interested in the transient behavior of the jet as it propagates outwards, and this for a wide variety of beam densities and Mach numbers.These questions have received very little attention experimentally because of the difficulty of producing pressure-mathced jets, particularly those of low enough density and high enough Mach numbers to be relevant to astrophysical jets. Finally, the "far from the nozzle" behavior of supersonic jets has not been addressed in enough detail experimentally; therefore, we summarize here what we have learned on the basis of our ongoing program of numerical simulations. A comprehensive overview of our program and results can be found in Smarr, Norman, and Winkler (1983).

Our physical model, numerical methodology, and some early results were reported in Norman et al. (1982, hereafter Paper I). Since then, we have extended our study to include a wider variety of jets (density ratios and Mach numbers), and these for longer evolutionary times and hence lengths. The propagation and morphology of the new parameter set are described in Norman, Winkler, and Smarr (1983, hereafter Paper II). Distinguishing characteristics of the numerical simulations are their high resolution ($\sim$ 50,000 zones) and the use of a 2-fluid Eulerian hydrodynamics code that allows us to study the detailed behavior of fluid instabilities of the boundary between jet and ambient gas (Norman 1980; Norman and Winkler 1984). This has been decisive for elucidating fluid dynamical mechanisms for the produc-

tion of knots in pressure-matched supersonic jets.

We find that two qualitatively different mechanisms operate to produce knots in pressure-matched axisymmetric supersonic jets. Both mechanisms create local knots of high pressure by exciting large-scale systems of shockwaves in the supersonic beam. These two mechanisms may operate together, individually, or not at all, depending on the parameters of the jet and its evolutionary age or length.The two parameters are η, the density ratio of the undisturbed beam gas to the undisturbed ambient gas, and M, the Mach number of the beam defined as the ratio of the beam's inflow velocity with respect to the undisturbed gas to its internal sound speed. All calculations were made assuming an ideal gas equation of state for both jet and ambient gas, with the ratio of specific heats in each gas equal to 5/3.

The existence of two distinct mechanisms is the result of a simple morphological trend described in Paper II, which is that the heads of diffuse beams ($\eta < 1$) are embedded in a thick cocoon of backflowing gas which "splashes back" from the advancing head of the jet, whereas all dense beams ($\eta > 1$) are essentially "naked". The dividing line between the two regimes is not sharp, and is indicated only approximately in Figure 6. The boundary's dependence on Mach number reflects a second morphological trend, namely that a high Mach number diffuse beam supports an extensive (in length) cocoon, whereas a low Mach number diffuse beam supports only a lobe in the vicinity of the working surface.

Knots in the cocoon-dominated regime (cf. Fig. 6) are produced behind oblique shockwaves which are excited in the beam by perturbations arising in the cocoon. Perturbations are caused either by Kelvin-Helmholtz instability of the cocoon/ambient gas boundary (see Paper I, Fig. 13a), or by flow dynamics within the cocoon itself (cf. Fig. 7). The dynamics of the working surface of a diffuse jet (i.e., unsteadiness due to the Rayleigh-Taylor instability and vorticity generation at shock triple points) ensures that the cocoon will always dynamically perturb the central supersonic beam. This is illustrated for the case shown in Figure 8.

Figure 8a shows the shock/rarefaction structure of the jet with a flow visualization technique analogous to that described for the kinematic structure plots in Paper II. Large positive and negative

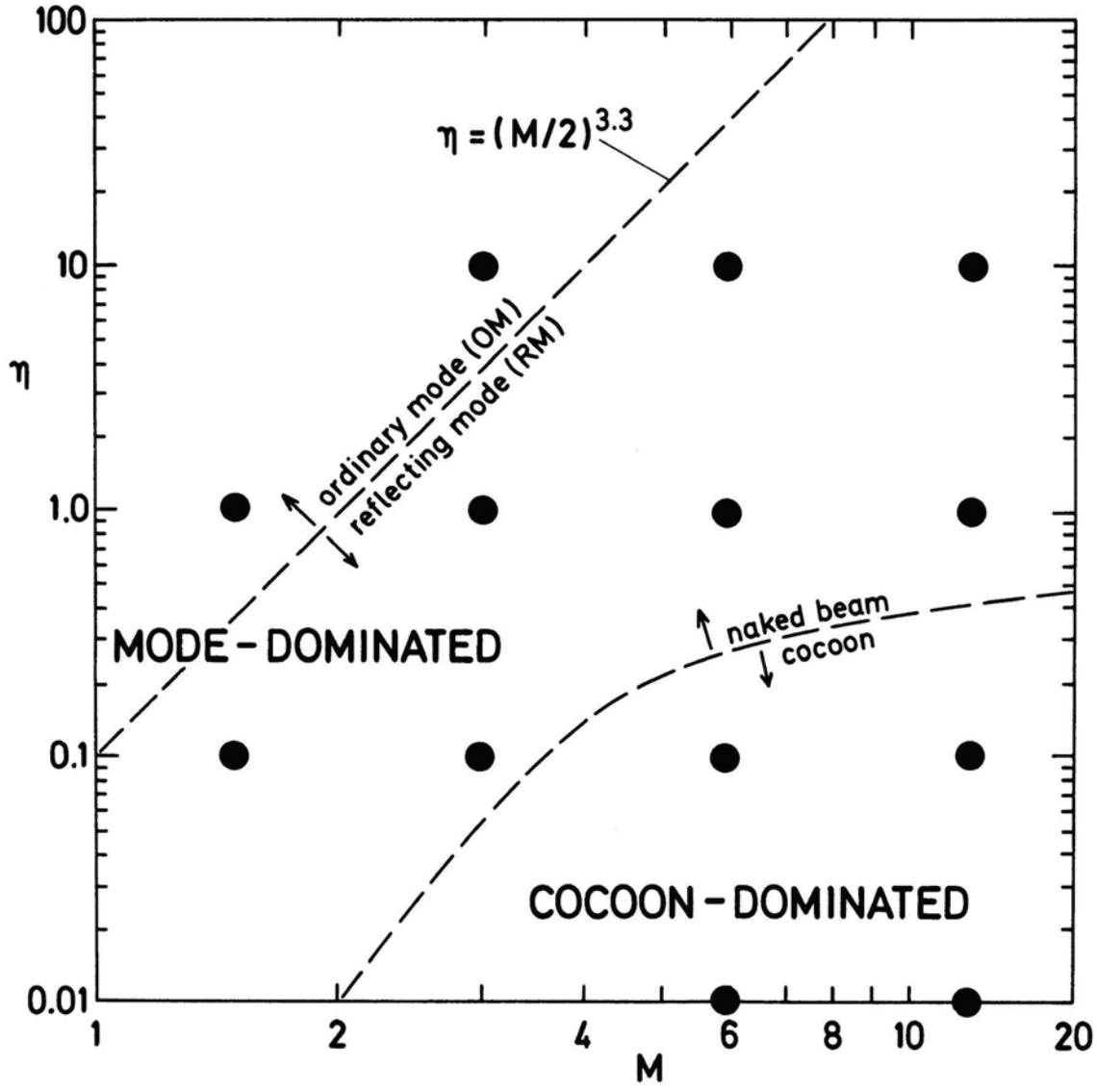


Fig. 6. - Regions in the (η,M) plane showing the dominant causes producing internal shockwaves in pressure-matched supersonic jets, as determined numerically. Black dots indicate 13 cases computed. Shocks in diffuse beams ($\eta < 1$) are excited by motions in the surrounding cocoon (cocoon-dominated, cf. Fig. 8). Jets without cocoons (naked beams) lie above the lower dashed line (approximate), and develop internal shocks through the nonlinear growth of pinching modes of the Kelvin-Helmholtz instability (mode-dominated, cf. Fig. 9). Perturbation theory distinguishes between the ordinary mode (OM), and reflecting modes (RM) (Ferrari, Trussoni, and Zaninetti 1981a; Cohn 1983). Cohn's boundary is given by the upper dashed line.

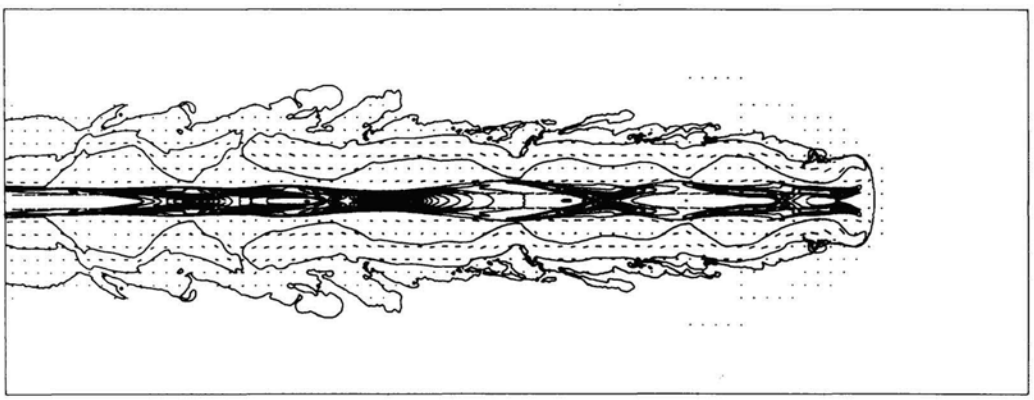


Fig.7. - Contour plot of kinetic luminosity ($\rho v_{\parallel}^3$) and velocity vectors in a cocoon-dominated jet (η = 0.01, M = 12), showing how internal shocks are excited in the beam by flow dynamics in the cocoon. Vectors are drawn at every 8th zone.

flow divergences $\nabla \cdot v$ identify strong rarefaction and shock waves, respectively, and are indicated by the light and dark grey regions within the jet boundary shown in black. Downstream of each shockwave are produced the high-pressure knots and hotspots shown in Figure 8b. Pressure values are represented by fifteen shades of grey ranging between the minimum (black) and maximum (white) pressure in the flow. In general, the maximum pressure exceeds the ambient pressure by a factor of M^2.

The dynamic nature of the knots thus produced is shown in Figure 8c. Here, pressure on the axis of the jet is plotted as a function of space (horizontal axis) and time (vertical axis) for the duration of the jet's computed evolution. For orientation, the space-time trajec-

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Fig. 8. - Shock structure, strength and history of a cocoon-dominated jet (η=0.01, M=6). a) Composite jet structure showing: shockwaves (dark grey), rarefaction zones (light grey), jet gas (white), external gas (dark grey), and contact discontinuity (black). b) Grey plot of gas pressure where white is maximum and black is minimum. Note the formation of high-pressure knots behind intersecting shockwaves in beam. c) Grey plot of pressure on the axis of the jet as a function of space (horizontal axis) and time (vertical axis) records strength and motion of working surface shock(s) and beam shocks. P_{max}=92, P_{min}=1.4x10^{-2} (P_{amb}=1).

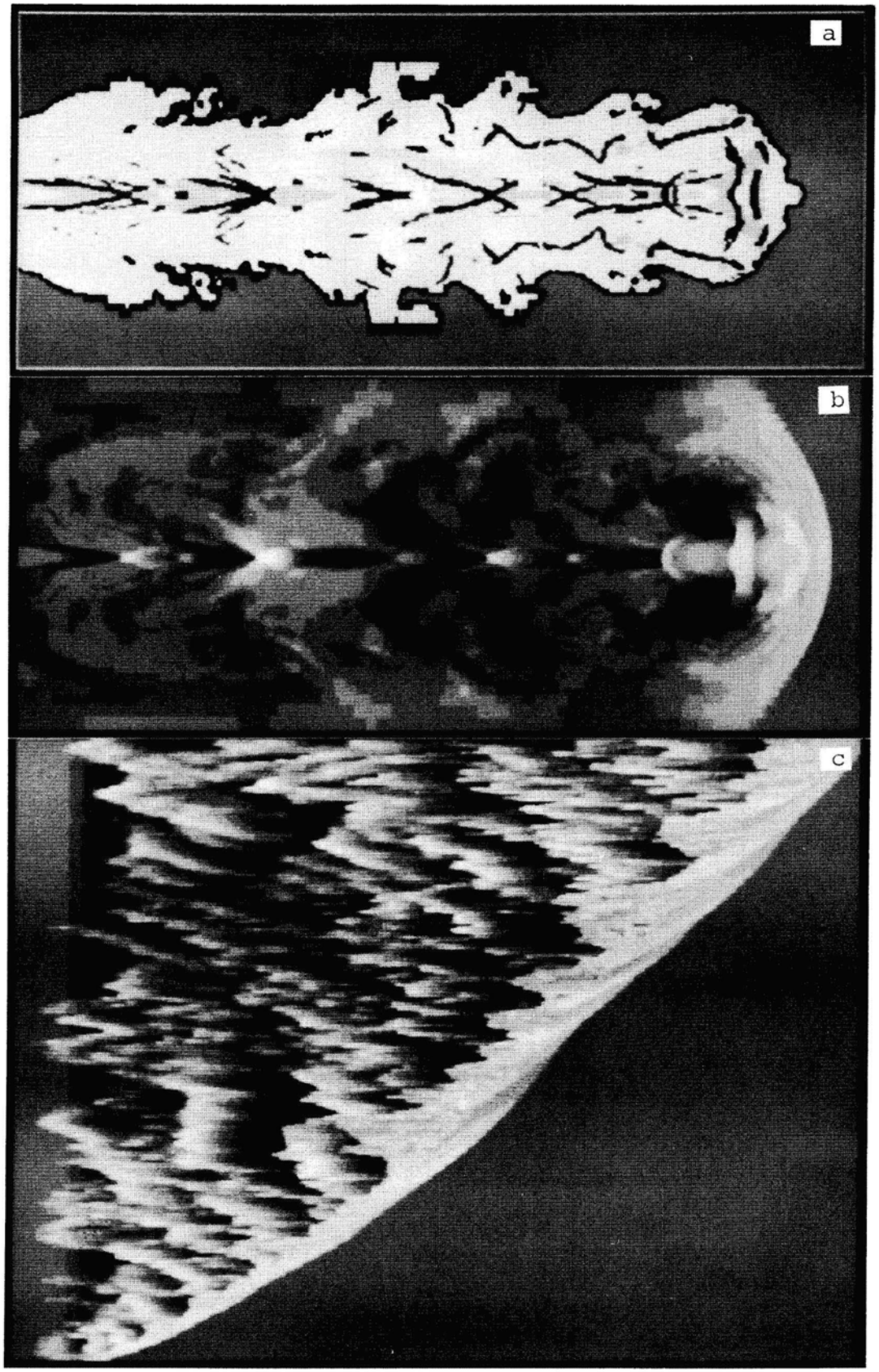


Fig. 8

tory of the head of the jet is from lower left to upper right, and can be seen to be a decelerating one. Variations in the pressure at the working surface derive from the previously mentioned unsteadiness of the contact discontinuity, and produce the negative velocity high-pressure features (knots) seen moving to the left. Knot velocity is limited to the cocoon backflow velocity; knot pressure can thereby exceed the naively drived ram-pressure value $\rho_{beam} v^2_{beam}$ due to the higher relative velocity between beam gas and shock. The existence of strong rarefaction waves immediately preceeding the strong shocks leads to pressure jumps of 3-4 orders of magnitude within the beam.

Knots in cocoon-dominated jets have a rather disordered strength and distribution history due to the rather chaotic nature of the driving mechanisms. On the other hand, we find that jets without cocoons (naked beams) develop knots via the nonlinear growth of specific modes of the Kelvin-Helmholtz instability of the beam/ambient gas boundary. These unstable modes then saturate through shock formation, leading to knots which are ordered in their spacing and strength. Figure 9 shows an example of a mode-dominated knotty jet.

We can see from Figures 9a and 9b that this $\eta = 0.1$, $M = 3$ beam has developed a regularly spaced pattern of alternating shock and rarefaction waves. The shocks have the characteristic X shape (bi-conical considering the assumed axisymmetry) encountered in the oscillation of an underexpanded supersonic jet (cf. Fig. 3b). The mode's wavelength is approximately twice the mean jet diameter, again reminiscent of the behavior of laboratory supersonic jets in the saturation limit (cf. Section III). Notice, however, that in contrast to the laboratory situation, the shocks are not being excited by a large pressure imbalance at the inlet, but rather grow in strength as the flow proceeds downstream. This observation argues that the shocks are the result of a physical instability which saturates at finite amplitude. The high-pressure knots behind each shock intersection bear a remarkable similarity to those in Figure 3 as well, except for one thing: as Figure 9c shows, this is a propagating mode, whereas the pattern of shocks and knots in Figure 3 is stationary. As can be seen in Figure 9c, the pattern velocity is comparable to the velocity of the head of the jet, which is also comparable to the sound speed

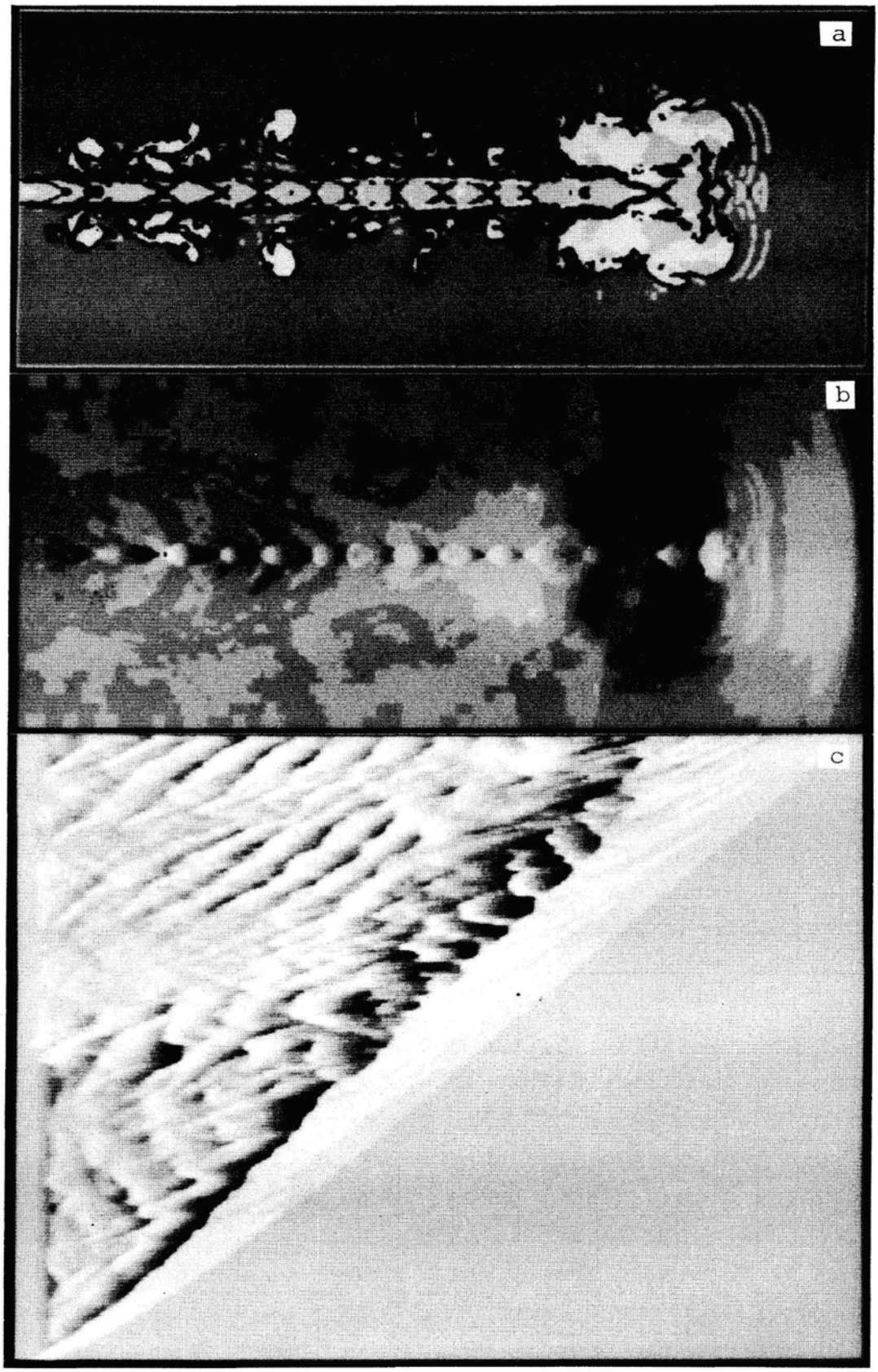


Fig. 9. - Shock structure, strength and history of a mode-dominated jet (η=0.1, M=3). a) Jet structure showing multiple internal shock-waves. b) Grey pressure plot showing periodic high-pressure knots. c) Space-time plot of gas pressure on the axis showing development of ordered mode. P_{max}=7.5, P_{min}=1.5x10^{-2}.

in the external medium. Thus, the amplitude and pattern velocity of the instability seem to be limited by the formation of external shockwaves. What is this propagating mode, and what is its relation to the mode excited in laboratory jets?

To answer this, we studied several "clean cases" - initially homogeneous cylindrical beams with vortex-sheet outer boundaries (i.e., sharp discontinuity). The perturbation theory of Kelvin-Helmholtz instabilities in such beams has recently been developed (Hardee 1979; 1982; 1983; Ferrari, Trussoni, and Zanietti 1978; 1981a; Ray 1981; Cohn 1983), and we hoped to make an identification on this basis. An important discovery by Ferrari et al. (1981a) was that pinching modes come in two varieties: the well-known ordinary mode (OM), which is important at low Mach numbers ($M \lesssim 2$), and an infinite spectrum of so-called reflecting modes (RM), at least one of which is important at any given Mach number $M \gtrsim 2$. Reflecting modes derive their name from the special property of a supersonic shear surface investigated by Miles (1957), which is that for certain angles of incidence, a plane linear wave (sound wave) will be reflected from the surface with a greater amplitude than it had upon incidence. A reflecting mode instability relies on the existence of two supersonic shear surfaces in close proximity, such as in a supersonic beam, between which waves can repeatedly reflect. One can imagine that after enough amplifying reflections, an initial wave of linear amplitude could grow to nonlinear amplitude and disrupt the flow in the beam. Ferrari et al.'s linear analysis showed that the reflecting modes' dominant wavelength is of the order of the beam circumference, independent of Mach number. Cohn (1983), in a similar analysis, emphasized the importance of the RM's for jets of astrophysical interest especially regarding knot formation, and divided the (η,M) plane into OM and RM dominated regimes. Cohn's boundary is redrawn on our parameter space of models in Figure 6.

We computed two clean cases on either side of the boundary with parameters (η = 1, M = 1.5 and 3). Small perturbations were applied at the beam inlet. Results are shown in Figure 10. We see in Figure 10a that perturbations of the Mach 1.5 beam grow to become large-amplitude oscillations of the beam boundary which neck down the flow.

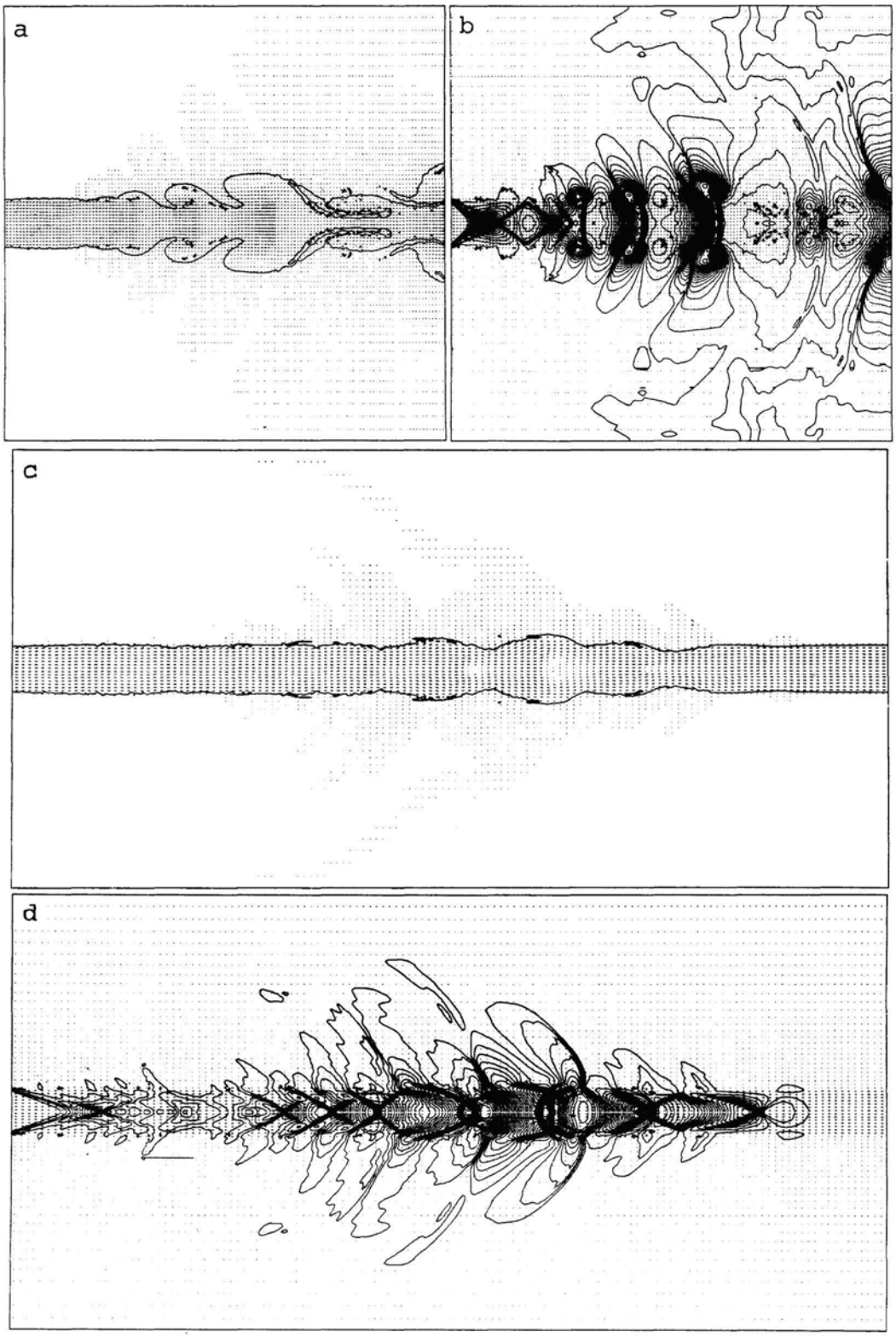


Fig. 10. - Structure of pinch instabilities in the nonlinear regime and their dependence on Mach number. a,b) η=1, M=1.5; c,d) η=1, M=3; a,c) Beam boundary and velocity vectors (drawn at every 3rd zone); b,d) Pressure contours, showing planar shocks in OM regime and bi-conical shocks in RM regime.

Planar shockwaves form just upstream of the pinched regions (Figure 10b). Figure 10c shows that the undulations of the Mach 3 jet's boundary are less severe, and are coupled to the network of criss-crossing (biconical) shockwaves in the beam seen in Figure 10d. The former mode is seen to be more disruptive than the latter for two reasons. First, the large amount of entrainment in the M = 1.5 case decelerates the bulk flow through momentum sharing with the entrained gas. Second, the planar shockwaves standing upstream of the constrictions make the flow locally subsonic, whereas in the M = 3 case the flow remains wholly supersonic. This is because in an oblique shockwave, only the component of velocity normal to the front is rendered subsonic.

Thus, we observe a change in the character of the nonlinear flow which coincides with the change in modal structure as predicted by the linear theory. On this basis, we tentatively identify Figures 10a,b with the nonlinear development of the OM, and Figures 10c,d with the nonlinear development of the first RM. There are also some suggestive similarities between the transverse structures of the modes in both the linear and nonlinear solutions which point to this conclusion. Nevertheless, we feel that additional detailed studies are required in order to substantiate this identification. Therefore, in order to avoid confusion with the linear modes, we shall refer to the nonlinear modes displayed in Figure 10 by the shock structures they produce, i.e., planar (Fig. 10b) and biconical (Fig. 10d).

Returning to Figure 9, we identify the series of biconical shockwaves as qualitatively the same nonlinear mode which manifests itself in Figures 10c and 10d. This is made plausible by noting how near the two cases are in parameter space (cf. Fig. 6), and that the cocoon of the jet in Figure 9a has quite mixed with the ambient gas, leaving the beam naked. At advanced times, the two calculations are therefore quite similar, although the initial conditions were quite different.

We find regularly spaced biconical shocks and high pressure knots in all the cases we computed which lie between Cohn's reflecting mode boundary and our cocoon boundary in Figure 6. We observe that the growth-length scales roughly with the Mach number, yet the

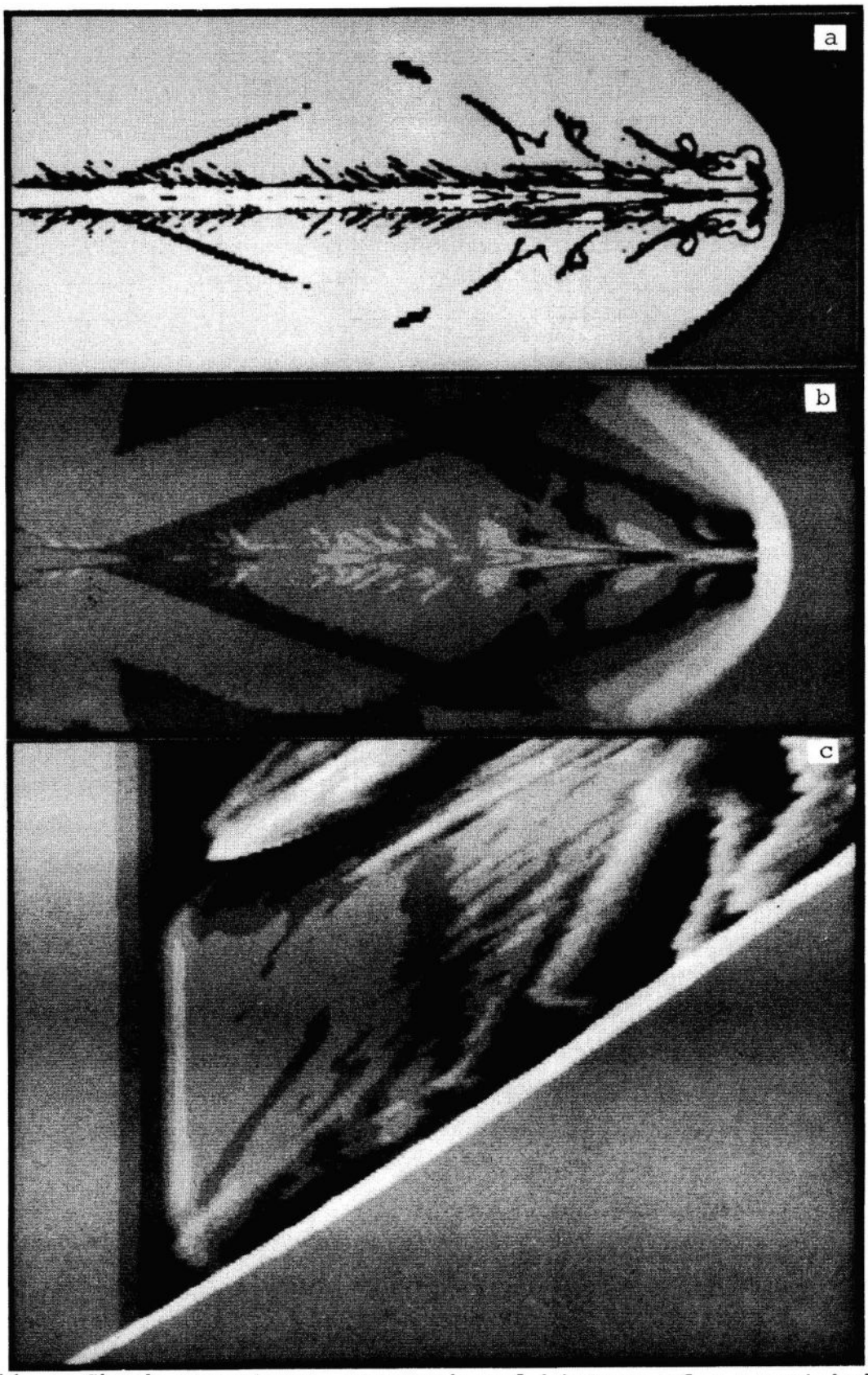


Fig. 11. - Shock structure, strength and history of a quasi-ballistic jet (η=10, M=12). a,b) Note relative absence of internal shocks despite impingement of spurious waves which reflect from boundaries. c) Space-time pressure plot showing that the working surface hotspot is the dominant feature.

wavelength of the mode when fully saturated is rather insensitive to the Mach number ($2.3 < W/D < 2.6$). This behavior is consistent with the properties of the linear reflecting pinching modes (Ferrari et al. 1981a; Cohn 1983), and further suggests that an intimate relationship exists between the RM and the biconical mode. Again, careful quantitative studies must be undertaken to establish this connection convincingly.

The scaling of growth length with Mach number means that a jet which is shorter than one growth length of the dominant mode will have no large-scale internal shocks and hence no knots (assuming nonaxisymmetric modes do not excite them). An example of this is shown in Figure 11. An examination of the three modes of display reveals very little shock dissipation in the beam until it encounters the working surface shock at the end of the jet. At $M = 12$, the growth length is at least 24 beam diameters (one round trip of the sound wave across the beam), which is only slightly less than the length of the jet in Figure 11 (35 beam diameters). However, if we continue the evolution to greater lengths (cf. Fig. 12), we indeed find that the biconical mode grows to large amplitude.

The two cases we computed which lie in the ordinary mode regime (cf. Fig. 6) were found to develop the pinching mode displayed in detail in Figures 10a and 10b. The jets were rapidly disrupted

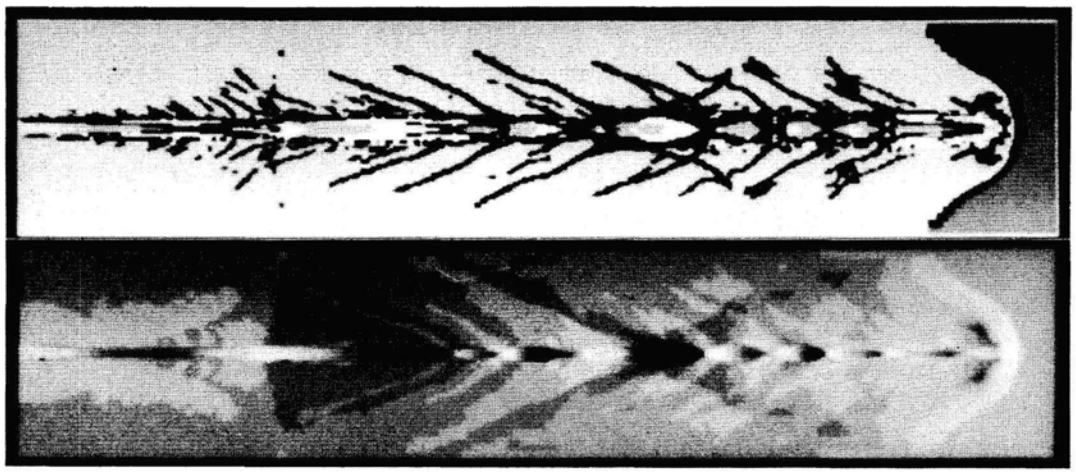


Fig. 12. - a) Development of internal shocks, and b) high-pressure knots due to biconical mode in a high Mach number jet ($\eta = 10$, $M=12$).

through the entrainment of external gas and the formation of planar shockwaves, as described above. Therefore, one of the major results of our numerical investigations is that jets unstable to the planar shockwave mode cannot propagate supersonically to large distances, unless they are exceedingly dense and hence essentially ballistic. On the other hand, jets on the right side of the OM/RM boundary may have strong internal shocks with very little effect on the jet transport.

V. KNOTS IN NONAXISYMMETRIC JETS

It has become customary to consider knots and wiggles to be two independent classes of small-amplitude structures in the general morphology of astrophysical jets (e.g. Trussoni, Ferrari, and Zaninetti 1983). This division derives partly from perturbation theorists' attempts to explain knots and wiggles on the basis of two independent modes of instability for supersonic beams - the pinching and helical modes, respectively (e.g. Hardee 1983; Ferrari et al. 1983). The helical mode is but one special case in a general class of modes involving bends. Likely sources of perturbations which may excite nonaxisymmetric modes are precession and/or orbital motion of the jet-producing source, and gradients, inhomogeneities, and cross-winds in the ambient medium through which the jet is propagating. We shall show, on the basis of a numerical example, that the nonlinear development of a bend in a supersonic flow also produces shock-excited knots, due to kinking at the bends. At a kink, a significant fraction of the beam power is shock-thermalized, resulting in high-pressure knots of geometrical character and distribution different from those produced by the nonlinear biconical pinch instability of the last section.

We model helical modes in a 3-d cylindrical beam by studying kinking modes in a 2-d infinite slab. Ferrari et al. (1982) found that the two systems have entirely analogous instability properties in the linear regime; a nonlinear comparison will have to await 3-d numerical simulations. We proceed as with the cylindrical jets, continuously introducing slab gas at the left in Figure 13a. Again,

pressure balance between the incoming jet and undisturbed medium is assumed. To simulate a helical perturbation, we wiggle the slab at the inlet through $\pm$ 0.05 radians with a frequency corresponding to half the fundamental wavelength $\lambda_f \equiv 2W\sqrt{M^2-1}$; W is the slab width. Physically, λ_f is the distance downstream from a disturbance which is required for a signal, propagating at the Mach angle $\theta_M = \sin^{-1}(1/M)$ with respect to the slab boundary, to cross the slab, reflect, and return.

The smooth bend becomes a kink in one wavelength (cf. Fig. 13b), exciting shockwaves both inside and outside the slab. The internal shocks are those required to deflect the supersonic flow through a finite angle at the kinks; the external shocks are driven by the supersonic advance of the kinks through the external medium. The latter were first discovered in the numerical calculations of Tajima and Leboeuf (1980) and investigated more recently by Nepveu (1982) and Woodward (1982, 1984). The shocks yield the characteristic alternating pattern in the pressure field shown in Figures 13c,d.

Such behavior in astrophysical jets may have interesting observational consequences. First, the existence of shockwaves in both media may give rise to different sorts of emission (here we should point out that nonlinear pinch instabilities also drive shocks into the external medium, cf. Fig. 10). The knotty nonthermal emission in galactic jets would presumably be excited by the internal shockwaves, while the observed line emission (e.g. Miley 1983) could be excited by the external shockwaves. In stellar jets, shock-excited gas both inside and outside the jet could be contributing to the characteristic emission line spectra observed.

This picture contrasts with Benford's (1981) for the bright knots in the M87 jet, in which he did not consider the internal shocks, but rather suggested that the external "bow shocks" were responsible for in situ particle acceleration and thereby knotty synchrotron emission. Conceivably, both shocks may help in the acceleration process, especially for the highest energy particles, since the shocks are physically so close. However, the relative amounts of synchrotron emission coming from inside and outside the beam depend on the local magnetic field strengths, which may be vastly different and favor the jet interior.

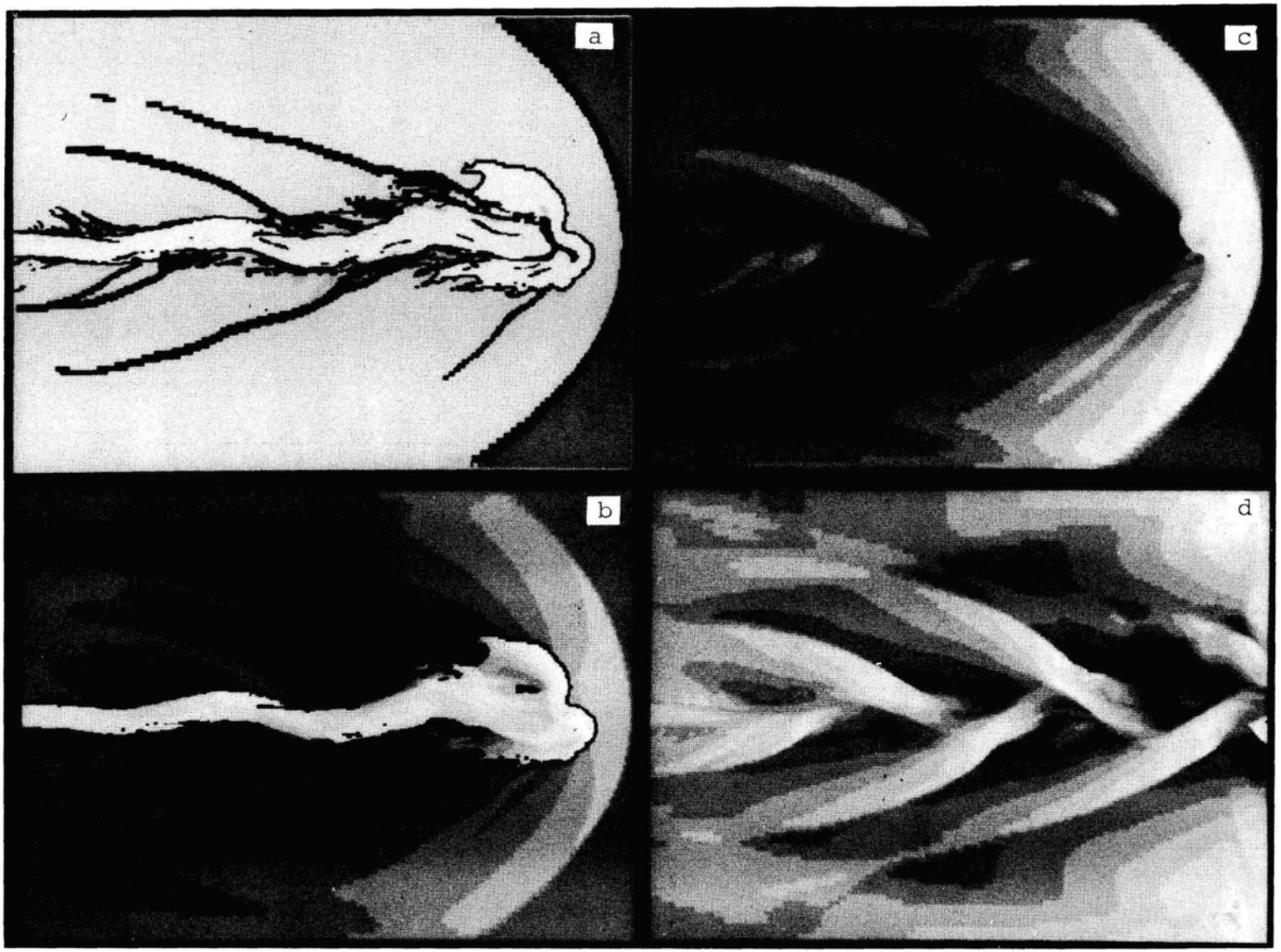


Fig. 13. - Kink instability in a slab-symmetric supersonic jet (η=10, M=12), excited with a 5% perturbation applied at the inlet: a) shock structure; b) grey plot of density; c,d) grey plots of pressure at two evolutionary times, showing the development of high-pressure features at kinks with characteristic side-to-side distribution.

As a second observational consequence, kink-related knots may be far apart yet regularly spaced, unlike the pinch-related knots discussed in Section IV. Woodward (1984), in a series of high-resolution numerical simulations of kink instabilities in slabs, has determined that the fundamental odd mode with wavelength $\lambda \approx \lambda_f$ is the fastest-growing odd mode. High Mach number jets would accordingly be expected to produce kinks with a wavelength $\lambda \approx 2MD$.

Finally, in the region where a sinusoidal mode attains nonlinear amplitude, one should see increasingly brighter knots with a more pronounced side-to-side distribution. The fine structure of the knots should be elongated in the direction of the bend, and become increasingly oblique to the jet axis as the kinks reach large amplitude. Thus, knot spacing, lateral distribution, brightness, and fine structure may all be physically related to the underlying flow variables via a simple fluid dynamical model of kinking supersonic beams.

VI. APPLICATIONS AND OUTLOOK

In Section III-V we have described a number of fluid-dynamical mechanisms which induce large-scale systems of shockwaves in supersonic jets. The effect of these embedded shockwaves is to create local regions of high energy density in the flow which would be observed as knots of enhanced emission if these mechanisms were operative in astrophysical jets.

The idea of using shockwaves to excite emission knots is not new; both Rees (1978) and Blandford and Königl (1979) proposed shock models for the knots in the M87 jet (cf. Fig. 1a). However, there is an important qualitative difference between the present mechanisms and these previous theories; namely, that the system of shockwaves and knots produced by mechanisms described herein reflect local properties of the jet flow, and not properties of the jet source as in the time-variability theory of Rees (1978), nor do they reflect the earlier and perhaps far-upstream history of the jet as in the cloud-entrainment model of Blandford and Königl (1979). As such, there is the hope and the possibility that the observed properties of emission knots, such as their spacing, lateral distribution, motion,

brightness, and fine structure, may allow us to infer the physical parameters of the underlying flow. This is particularly important in the case of extragalactic jets, where these quantities are especially hard to determine.

The basic approach of attempting to infer flow parameters of radio jets on the basis of observed small-scale morphologies (i.e., knots and wiggles) was perhaps first clearly expressed by Hardee (1979), who interpreted these morphologies with the aid of linear Kelvin-Helmholtz instability analyses of supersonic beams. Recent attempts of this nature are given in Ferrari et al. (1983) and Hardee (1983). The numerical gas-dynamical simulations described herein and in Woodward (1982, 1984) extend this approach and complement the linear analyses by addressing and answering questions that are outside the realm of perturbation theory, such as determining the dominant mode of instability in a given situation, and characterizing its structural, propagation, saturation, entrainment, and disruption properties in the nonlinear regime. Before considering an astrophysical application of the knot-producing mechanisms described in the previous sections, we summarize the results of our numerical investigations on Kelvin-Helmholtz instabilities in supersonic beams and slabs.

1) Pinching modes in the OM regime (cf. Fig. 6) disrupt the beam when they become nonlinear through extensive entrainment of external gas and the formation of planar shockwaves within the beam normal to its flow direction (cf. Fig. 10a,b).
2) Pinching modes in the RM regime (cf. Fig. 6) saturate by forming large-scale biconical shock systems (cf. Fig. 10d) which have peak strength in the center of the jet. Assuming knot emissivity scales with some positive power of the energy density, center-brightened knots are produced.
3) Biconical shockwaves can be excited at high Mach numbers (cf. Fig. 12), with a spatial growth-length which increases roughly linearly with the Mach number.
4) In a constant or near-constant pressure background, the biconical shockwaves produce quasi-periodic knots with a spacing of 2-3 beam diameters. The knots move with a pattern velocity in excess of the external sound speed, but limited by the advance velocity of the

head of the jet.

5) Knot-interknot and knot-ambient pressure ratios increase with the Mach number.
6) Biconical pinching modes are less disruptive than the ordinary mode (cf. Fig. 10c), allowing the jet to propagate to large distances assuming it is of high Mach number (cf. Fig. 12).
7) Wiggles growing from linear perturbations in a high Mach number jet will be of long wavelength ($\lambda \approx 2MD$, Woodward 1984) and spatial growth-length.
8) Wiggles in the nonlinear regime become kinks and excite internal and external shockwaves, which could excite knots and diffuse emission (cf. Section V).

We reiterate that the relationship between linear reflecting modes and nonlinear biconical shock systems is not completely understood. Therefore some of our conclusions are partly conjectural, and provide a framework for further study.

As an application of the knot-producing mechanisms discussed above, we present an interpretation of the detailed structure of the radio jet in NGC6251 as mapped with the VLA by Bridle and Perley (1983), and exhaustively discussed by Perley, Bridle, and Willis (1984, hereafter PBW). The total intensity contours in Figure 14, taken from their work, show this object to be prototypical of a morphological class first addressed by Hardee (1979), and to which the jets in M87, 3C111, 3C465, 4C32.69 and others seem to belong - namely, a long straight jet exhibiting well-developed knots close to the compact source, yet deviating significantly from straightness only much farther out.

This segregation of the jet into axisymmetric and nonaxisymmetric segments (PBW) is difficult to explain with a simple application of the perturbation theory of fluid instabilities if one assumes: a) the ordinary pinching mode is responsible for knots, b) the helical mode is responsible for wiggles, c) the beam is of constant Mach number, and d) all modes grow from linear amplitude. The difficulty is that under these circumstances, the helical mode will always dominate the pinching mode (ordinary) except for Mach numbers of about unity (Hardee 1979). Accordingly, a long straight jet without inner knots would result at high Mach numbers, or a short straight

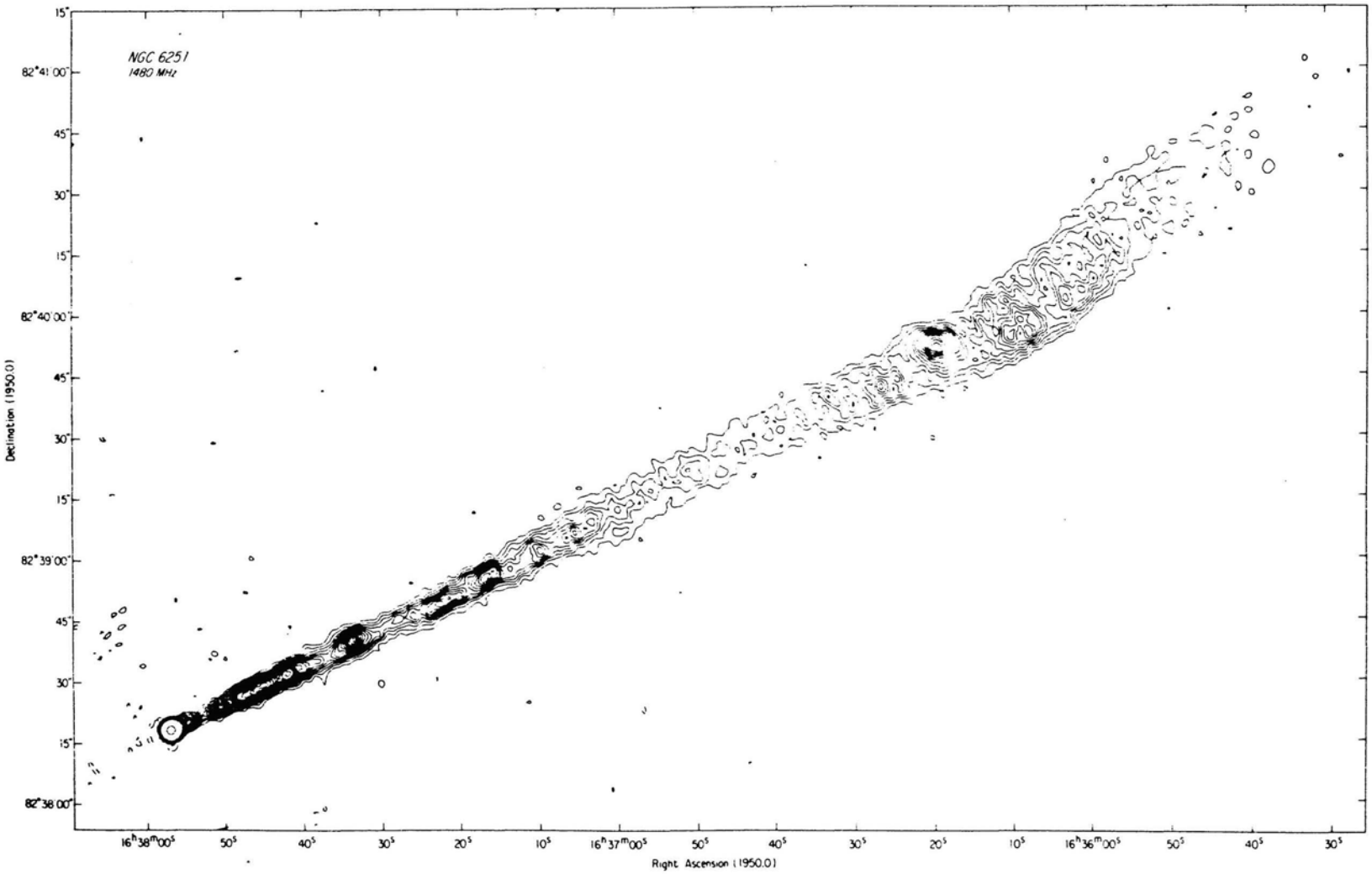


Fig. 14. - Total intensity contours of radio jet in NGC6251 mapped at 1.48 GHz with the VLA (Bridle and Perley 1983), showing a long straight jet with well-developed inner knots and outer wiggles.

jet with inner knots would result at low Mach numbers. Hardee's remedy was to invoke low Mach numbers near the source to allow the pinching mode to dominate, followed by a rapid acceleration to high Mach numbers which would convect the helical perturbation far down the jet before it could grow.

PBW derive a Mach number of order 14 for the innermost 8 kpc (20") of the jet, ruling out such an interpretation. We can also rule out Hardee's interpretation on the grounds that, as we have shown, the OM pinching mode is a disruptive instability. Clearly, what is needed is a pinching mode which is competitive with kinking or helical modes at high Mach numbers, yet one which does not disrupt the flow, and a mechanism for preferentially exciting this mode.

With these considerations in mind, we suggest the following interpretation for the structure of the NGC6251 jet. Pinching modes which saturate as strong biconical shocks are responsible for the inner knots. These pinching modes are excited with nonlinear amplitude via the jet reconfinement mechanism of Sanders (1983). Jet widening and/or mode damping is responsible for the middle fading region (PBW). A kink mode, which grows from a small initial amplitude, is responsible for the outer wiggle ($\theta > 240"$, PBW) and associated emission knots as it grows to finite amplitude. The jet is of constant or slowly increasing Mach number which is large enough ($M > 10$) to allow only a few growth-lengths of the kink mode to fit between the reconfinement shoulder and the outher wiggle. Implicit in our interpretation is that the emission features reflect the underlying flow structure. This is a natural consequence of the shock model, although many details have yet to be worked out.

Jet reconfinement is a natural mechanism for exciting the axisymmetric mode with nonlinear amplitude, which we require to produce the observed segregation of axisymmetric and nonaxisymmetric parts of the jet. As discussed by Sanders (1983), a supersonic jet propagating down a pressure gradient steeper than r^{-2} will come out of pressure equilibrium with its surroundings and eventually become free. Should the free jet encounter a plateau in the ambient pressure which is decreasing less rapidly than r^{-2}, it will be reconfined and generate a convergent shock in an entirely analogous manner to underexpanded laboratory jets. Sanders was able to fit the shape of the

reconfinement shoulder in NGC315 with this model, and made the important observation that the position of an emission knot produced by the reconfinement shock would be related to the gross structure of the flow. Significantly, PBW report the existence of a strong emission knot in the NGC6251 jet whose location correlates with the observed position of the reconfinement shoulder as would be expected from the Sanders model.

How can numerical simulations help verify the model proposed above, and what is their role in understanding the physics of astrophysical jets? Numerical simulations can play a role in modeling realistic global flow patterns that cannot be investigated experimentally because of unique physical effects and regimes that exist in these jets (e.g. relativistic speeds, pressure and temperature stratifications in the confining gas, extreme Mach numbers and density ratios). Although 3-d simulations with meaningful numerical resolutions are not feasible on current machines, much can be learned through parallel 2-d simulations in axial and slab symmetry. An important test of this approach would be to model the general morphology of the NGC6251 jet to see whether a segregation of even and odd modes in a high Mach number slab symmetric "jet" is possible. In axisymmetry, one could try to reproduce its average intensity distribution by building in simple but representative emission models based on shock particle-acceleration theory. If a satisfactory fit could be demonstrated, then it would provide support for both our interpretation of the NGC6251 jet and the utility of this "hybrid" approach.

Aside from detailed modeling, numerical simulation is useful for investigating physical mechanisms. The work described in this paper is an example of this approach. We have attempted to show how numerical simulation can be used to test the range and validity of a proposed mechanism, to elucidate the behavior and effects of a mechanism in the nonlinear and time-dependent regime, to search for systematic behavior in solution space, to act as a bridge between experiment and analytic theory, and to serve as a laboratory for discovering new mechanisms. However, if numerical simulations are really to do these things and address topics of current interest with

regard to astrophysical jets such as large-scale bends, vortex dynamics and turbulence, nonaxisymmetric instabilities, and magnetic effects, then 3-dimensional models are needed. This has not been possible up to now because of insufficient computer storage and speed. Next generation supercomputers will begin to allow such an undertaking, and those that are planned for the second half of the 1980's should make high-resolution 3-d modeling a reality.

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