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# MECHANISM FOR INDUCING SYNCHRONOUS ROTATION AND SMALL ECCENTRICITY IN CLOSE BINARY SYSTEMS*

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## ABSTRACT

Rotation through a tidal bulge generates a fluid shear which, in stars, implies a very high Reynolds number: fully developed turbulence might result. Turbulent viscosities, synchronization, and circularization time scales are computed. The calculation contains no adjustable dimensional parameters. It explains the observed fact of synchronous rotation for orbital periods $\lesssim$4 days, nonsynchronous for larger periods. The observed small eccentricity of some short-period binary X-ray sources is also an immediate consequence.

*Subject headings:* binaries — hydrodynamics — interiors, stellar — turbulence — X-ray sources

A striking observational feature of close binary systems is that those with periods of $\lesssim$4 days have nearly circular orbits and have axial rotation periods which are approximately synchronous with their orbital periods (see Plavec 1970 for a review). Particularly remarkable are the binary X-ray sources like Cen X-3 or Her X-1 (Schreier 1975) since these must have had a large eccentricity resulting from the formation of their compact components (Lea and Margon 1973; Wheeler *et al.* 1974; Chevalier 1975). Zahn (1966*a*, *b*, *c*), Kopal (1968, 1972*a*, *b*), Alexander (1973), and others have developed an elegant formalism for treating the evolution of close binary systems due to the viscous tidal interactions of the two components. For late-type dwarf stars with convective outer envelopes, eddy viscosity due to the convection is large enough to explain rapid synchronization and circularization. However, most close binaries studied are B0–A5 spectral types which have only small convective cores and radiative outer envelopes. The demand that tidal friction be effective in the (short) main-sequence lifetime of these systems requires viscosities of order $\langle\mu\rangle \approx 10^{11}$–$10^{15}$ g cm$^{-1}$ s$^{-1}$ (Alexander 1973; Sutantyo 1974). By comparison, typical plasma or radiative viscosities in these stars are only $\sim 10^{2}$–$10^{4}$ g cm$^{-1}$ s$^{-1}$ (Kopal 1968). The possibility of magnetic viscosity has been invoked to explain this discrepancy (Sutantyo 1974), but (as Lecar *et al.* 1976 have also noted) one very natural mechanism seems to deserve further attention: the viscosity of fully developed turbulence driven by tidal shear.

The combination of a nonsynchronous rotation or an eccentric orbit with tidal distortion leads to a local fluid shear. The microscopic Reynolds number associated with this shear is many orders of magnitude larger than the critical value for the onset of fully developed turbulence, and the time scale for the establishment of this turbulence—the eddy turnover time—is astronomically short. Once turbulence sets in, the effective eddy viscosity can be estimated. Moreover, the result does not depend on the assumed magnitude of the microscopic sources of viscosity, as long as they are small enough to allow fully developed turbulence. We will see that the time scales for synchronization and circularization obtained have no adjustable parameters, and yet give good quantitative agreement with observations, in particular with the 4-day rule mentioned above.

We now give some details: In terms of a mean viscosity $\langle\mu\rangle$

$$\langle\mu\rangle \equiv \frac{9}{a_1^9}\int_0^{a_1}\mu(r)r^8dr\,, \tag{1}$$

Alexander (1973) gives the characteristic time scales, $\tau^{\rm circ}$ and $\tau^{\rm synch}$, for a star of mass $m_1$ and radius $a_1$ to change by order unity its eccentricity $e$ and rotation frequency $\omega_1$, respectively:

$$\tau_1^{\rm circ} = \frac{125}{121}(1-e^2)^5\left[\frac{A}{a_1}\right]^8\left[\frac{m_1^2}{m_2(m_1+m_2)}\right]\left(\frac{n}{\omega_1}\right)\left[\frac{m_1}{a_1\langle\mu\rangle}\right], \tag{2a}$$

$$\tau_1^{\rm synch} = \frac{75\alpha}{112}(1-e^2)^{9/2}\left(\frac{A}{a_1}\right)^6\left(\frac{m_1}{m_2}\right)^2\left[\frac{m_1}{a_1\langle\mu\rangle}\right]. \tag{2b}$$

Here $A$ is the semimajor axis of the relative orbit, $n = 2\pi/P$ where $P$ is the period, and $\alpha$ is the structure constant defining the star's moment of inertia:

$$I = \alpha m_1 a_1^2\,. \tag{3}$$

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Again following Alexander (1973), the tidally induced fluid shear can be calculated:

$$\sigma(r) \equiv \frac{1}{4\pi}\int_0^{\pi}\sin\theta d\theta\int_0^{2\pi}d\phi[\sigma_{ij}\sigma^{ij}-\tfrac{1}{3}\sigma_{mm}{}^2]^{1/2}=\frac{6\sqrt{7}}{5}\frac{m_2}{m_1}\frac{r^3}{A^3}n\delta_1\,, \tag{4}$$

where $r$ is the distance from the center of the star and $\delta_1$ is approximately $(\omega_1/n)-1$ or $e$, whichever is larger (shear comes either from rotation through a bulge or from a time-varying bulge amplitude). Since typical microscopic viscosities are in the range $\nu\equiv\mu/\rho\approx 10$–$10^3$ cm$^2$ s$^{-1}$, the Reynolds number is

$$\mathrm{R}=\frac{\sigma(r)r^2}{\nu}\gtrsim 10^{12}\gg \mathrm{R}_{\mathrm{crit}}\approx 10^5 \tag{5}$$

for masses of order $M_\odot$, lengths of order $R_\odot$, and times of order days. Moreover, $\mathrm{R}\gg\mathrm{R}_{\mathrm{crit}}$ even for $\delta_1\lesssim 10^{-3}$, which is observationally indistinguishable from $\delta_1=0$. Once turbulence sets in, the *effective* Reynolds number goes to some value $\mathrm{R}_T\approx 10$–$30$ (Tennekes and Lumley 1972, p. 134), in which case equations (5), (4), and (1) give a mean turbulent viscosity

$$\langle\mu\rangle_T\approx\frac{2m_2n\delta_1}{R_Ta_1}\left(\frac{a_1}{A}\right)^3\left[\frac{14}{m_1a_1{}^{11}}\int_0^{a_1}\rho(r)r^{13}dr\right]. \tag{6}$$

$\mathrm{R}\gg\mathrm{R}_{\mathrm{crit}}$ holds by such a large factor ($\sim 10^7$) that it remains true even if the relevant length scale is substantially less than the radius $r$. If turbulence is inhibited on this largest length (see below), then equation (6) must be scaled as the square of the largest length permitted. The term in square brackets we denote $K_\mu$; its value is roughly $K_\mu\approx 0.025$ (see below). With typical values of $m\approx 10\,M_\odot$, $a_1\sim 5\,R_\odot$, $\omega_1\approx 2\pi/4$ days $\approx 10^{-5}$ s$^{-1}$, $a_1/A\approx 0.3$, $\mathrm{R}_T\approx 20$, and $\delta_1\approx 0.1$, equation (6) gives

$$\langle\mu\rangle_T\sim 10^{12}\ \mathrm{g\ cm^1\ s^{-1}}\,,$$

just about what observations require. Note that except for $R_T$ (dimensionless and not too different from unity), only observable quantities have entered the calculation.

To be more specific, we can rewrite equations (2a) and (2b) using equation (6):

$$\tau_1{}^{\mathrm{circ}}\approx\frac{125}{242}\frac{R_T}{K_\mu\delta_1\omega_1}(1-e^2)^5\left(\frac{A}{a_1}\right)^{11}\left[\frac{m_1{}^3}{m_2{}^2(m_1+m_2)}\right], \tag{7a}$$

$$\tau_1{}^{\mathrm{synch}}\approx\frac{75}{224}\frac{R_T\alpha}{K_\mu\delta_1 n}(1-e^2)^{9/2}\left(\frac{A}{a_1}\right)^9\left(\frac{m_1}{m_2}\right)^3 \tag{7b}$$

*Example 1.* For AG Persei, using parameters given by Alexander (1973), and $\mathrm{R}_T=20$, $K_\mu=0.025$, $\delta_1=0.1$, $\alpha=0.2$ (see below), we obtain

$$\tau_1{}^{\mathrm{circ}}\approx 6.8\times 10^7\ \mathrm{yr}\,,\qquad \tau_1{}^{\mathrm{synch}}\approx 8.4\times 10^5\ \mathrm{yr}\,.$$

The synchronization time is consistent with the age of the system, $\sim 10^6$ yr. The circularization time may be consistent if the dimensionless constant multiplying these characteristic times is fairly small (there is some evidence in Alexander's numerical integrations that this is the case); alternatively, one may suppose that the system formed by fission with low eccentricity.

*Example 2.* For Cen X-3, with parameters used by Chevalier (cf. Schreier *et al.* 1972; Avni and Bahcall 1974), and $\mathrm{R}_T=20$, $K_\mu=0.025$, $\delta=0.01$, $\alpha=0.2$, we obtain

$$\tau^{\mathrm{circ}}\approx 1.7\times 10^6\ \mathrm{yr}\,,\qquad \tau^{\mathrm{synch}}\approx 1.9\times 10^6\ \mathrm{yr}\,,$$

also consistent with the age of the system.

We now calculate the period which should separate synchronous close binaries from nonsynchronous ones. A "typical" system has equal B5 components with $m\approx 7\,M_\odot$, $a\approx 4\,R_\odot$, and has an age of (on the average) half its main-sequence lifetime of $3\times 10^7$ years. Using $A^3=G(m_1+m_2)n^{-2}$, equation (7b) gives

$$P_{\mathrm{synch}}=4.0\ \mathrm{days}\left(\frac{\tau_{\mathrm{age}}}{1.5\times 10^7}\right)^{1/7} \tag{8}$$

[$\delta_1=0.1$, $K_\mu=0.025$, $\alpha=0.2$, $\mathrm{R}_T=20$], which agrees very well with observation. For massive X-ray binaries we have $a_1\approx 12\,R_\odot$, $m_1\approx 20\,M_\odot$, $m_2\approx 1.4\,M_\odot$, $\tau\approx 3\times 10^6$ yr. The period which should separate circularized systems from eccentric ones is

$$P_{\mathrm{circ}}=3.1\ \mathrm{days}\left(\frac{\tau_{\mathrm{age}}}{3\times 10^6\ \mathrm{yr}}\right)^{3/25} \tag{9}$$

[$\delta=0.1$, $K_\mu=0.025$, $\alpha=0.2$, $\mathrm{R}_T=20$], which is about what is observed. Note that the most plausible example of a noncircular massive X-ray binary, Vela X-1, has a period of 8.96 days; the orbit of this system is not, however, known conclusively (Avni and Bahcall 1975; Wallerstein 1974).

The turbulence that we posit should have significant effects on the envelope structure of stars in close binary systems. At radius $r$ in the star, the turbulent velocities would be of order

$$v_{\mathrm{turb}}\approx\sigma(r)r/\mathrm{R}_T\,,$$

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where $\sigma(r)$ is given by equation (4); the corresponding eddy turnover time is about

$$\tau_{\rm eddy} \approx R_T/\sigma(r) .$$

Until the degree of synchronization is quite precise, this is much shorter than the Kelvin thermal time scale for these stars, so that their interior temperature gradient ought to be nearly adiabatic rather than that given by radiative opacities. This should lead to systematic differences in the structure and in mass-radius-luminosity relations for even slightly nonsynchronous stars in close binaries, when compared with isolated stars or binaries which are precisely synchronous. The turbulent envelope should be roughly an $n = 3/2$ polytrope (whose numerical values of $K_\mu$ and $\alpha$ were used above) rather than the $n = 3$ polytrope which is normally a good approximation for, say, early B stars. By analogy with the case of convective envelopes, the internal structure may be sensitive to surface boundary conditions (Cox and Giuli 1968, § 20.6). One should calculate actual models for "turbulent main-sequence stars" and compare them to the data in the manner of Stothers (1973). Another consequence of turbulent mixing is that surface nuclear abundances may be abnormal: for example, N/C or N/O ratios may be higher than in isolated stars. Alternatively, the turbulent transport might not propagate through the many pressure scale heights separating photosphere and interior, and magnetic fields may also interfere.

Apart from magnetic fields, the principal theoretical uncertainty in this discussion is whether the huge formal Reynolds number of equation (6) actually does lead to turbulence on the scale, and with the eddy viscosity, estimated here. It may initially be difficult for turbulence to disrupt the stable stratification of a radiative star: Until the star is turbulently mixed, the gradient Richardson number can be positive and large. Once mixed, however, there is no inhibition to continued turbulence since the turnover time is so much faster than the Kelvin time. In axisymmetric stars it is possible to have differential rotation which has a large formal Reynolds number but which is nevertheless stable. The physical reason is that shear on a fluid element is not pressure-transmitted, and the motion is stabilized by gravitational and centrifugal forces. In the tidal-bulge situation, by contrast, a small fluid element experiences a time-varying shear force which is transmitted across its boundaries by pressure, and which forces it to change shape periodically. This is mathematically analogous to more familiar examples such as fluid flow past a periodic array of obstacles, well known to be turbulent at large Reynolds number. We have gone beyond the familiar examples, however, in assuming that the scale of the turbulence extends up to the size of the star (rather than, say, the size of the tidal bulge alone) and in assuming that the entire turbulent velocity of this largest scale is available to eddy viscosity. By contrast, Zahn's (1966*c*) discussion of convective eddy viscosity takes the relevant scale to be that smaller scale at which the eddy turnover time equals the period of the tidal shear. If the scale is in fact smaller than the size of the star, then the time scales of equation (7) are increased as the square of the length reduction factor. On the other hand, the effect on equation (8) is to change $P_{\rm synch}$ only as the 2/7 power, and $P_{\rm circ}$ only as the 6/25 power, so a fair agreement with observation may remain. Our less-than-conservative assumptions may turn out to be wrong, but they have some justification: Coriolis forces in the rotating frame can act to change eddy velocities on any scale by order unity in a time $\sim 2\pi/\omega_1$. If this time, rather than the turnover time, is the relevant time for loss of velocity "coherence," then the tidal shear—which *is* coherent over the whole star—should both drive turbulence on the largest scales, and work dissipatively on its full turbulent viscosity. We have begun a rigorous investigation of this question, which should generalize recent interesting work of Seguin (1976).

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