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Visualizing Black Hole Space-times

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In 1916 Einstein proposed a theory of gravity now known as general relativity. This theory provides the framework for much of our current understanding of some of the most unusual objects and events in our astrophysical universe, such as stellar collapse, black holes, neutron stars, quasars, gravitational radiation, and cosmology.

The equations describing the gravitational field are very complex. In their most general form they consist of a system of ten coupled hyperbolic-elliptic nonlinear partial differential equations, which would take many pages to write down. For this reason many implications of the theory have remained largely unexplored. Most of our knowledge about general relativity comes only from a few exact solutions, perturbative expansions about those solutions, and some important but general mathematical theorems that provide little detail about specific gravitational interactions.

Because it is difficult to obtain exact solutions for realistic situations with few or no symmetries, researchers more and more use high-performance computing hardware and software to construct accurate nonlinear time-dependent solutions to the gravitational field equations for multiple spatial dimensions.

Our research efforts concentrate on solving the Einstein equations numerically on supercomputers to study the interactions of one or more dynamical black holes with gravitational waves. This work is complemented by the scientific visualization professionals at NCSA, who have developed tools crucial to our understanding of numerically generated space-times. Through their efforts, we have produced scientific animations of gravitational wave and black hole space-times.

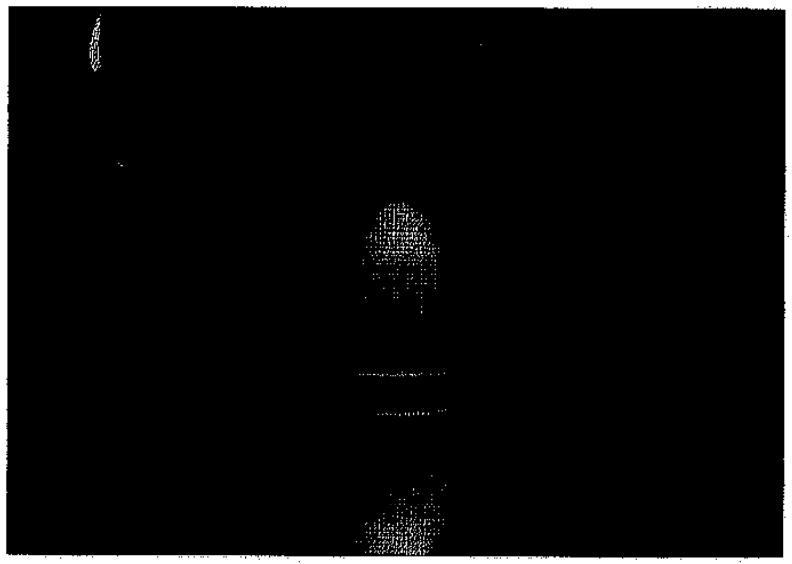


Figure 1. Embedding diagram and horizon history for the highly distorted single black hole at time $t \sim 2M$.

Black holes

The theory of general relativity predicts the existence of black holes. A black hole is a gravitational field so strong that nothing, not even light, can escape from its surface. All events taking place inside the black hole's two-dimensional surface (referred to as the event horizon) are invisible to observers outside the black hole.

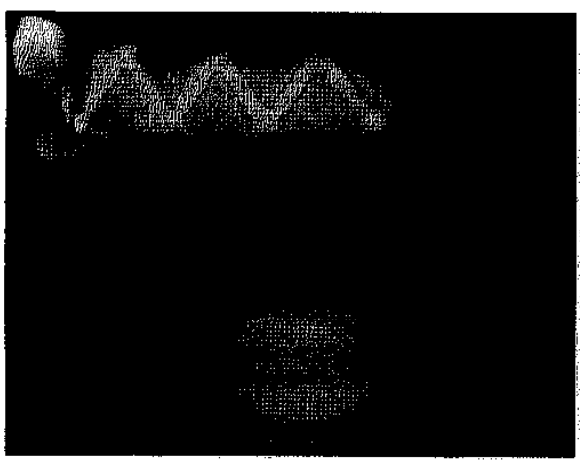


Figure 2. Same as Figure 1, except the embedding is shown at $t \sim 60M$.

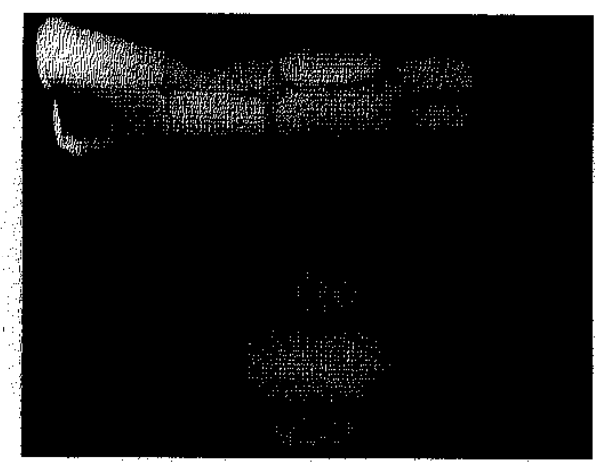


Figure 3. Embedding diagram and horizon history for the collision of two equal-mass black holes.

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In practice, it is difficult to locate event horizons from numerically generated space-times. Similar in concept to the event horizon, but easier to compute numerically, is the apparent horizon. Surfaces inside this apparent horizon are trapped inside the black hole, and the apparent horizon surface, if it exists, always lies inside or is coincident with the event horizon. The apparent horizon can provide clues to the behavior of the black hole geometry and to the amount of radiation possibly generated.

The standard textbook description of black hole space-times is usually limited to isolated "stationary" systems, such as the single spherically symmetric Schwarzschild black hole. This space-time is time independent. However, if the black hole is disturbed in some way, the system becomes dynamic and evolves according to the Einstein equations. Researchers know from perturbation calculations that a black hole perturbed away from its equilibrium spherically symmetric state will oscillate at its normal mode frequency (with a characteristic wavelength of 8.4 times the radius of the hole), just as a bell or stretched string will oscillate at its normal mode, or characteristic, frequencies when struck. Such systems slowly return to their undisturbed state as energy radiates away, by sound waves in the case of the bell or string, or by gravitational waves in the case of the black hole. (Gravitational waves are disturbances in the gravitational field that propagate with the speed of light through space-time.) The detection of the very specific waveforms we expect to emit from dynamic black holes would provide unambiguous evidence of their existence.

Visualizing black hole space-times

In this section we explore two different physical systems in detail. First, we generate a highly distorted single black hole space-time by placing a large amplitude gravitational wave in the vicinity of a black hole. Part of this wave gets swallowed up by the black hole, which causes the hole to oscillate at its normal mode frequency and emit radiation. Second, we investigate the head-on collision of two equal-mass black holes. The black holes are initially placed at a finite distance from each other, accelerate toward each other, and coalesce to form a single distorted black hole that eventually emits gravitational waves at the normal mode frequency of the final black hole.

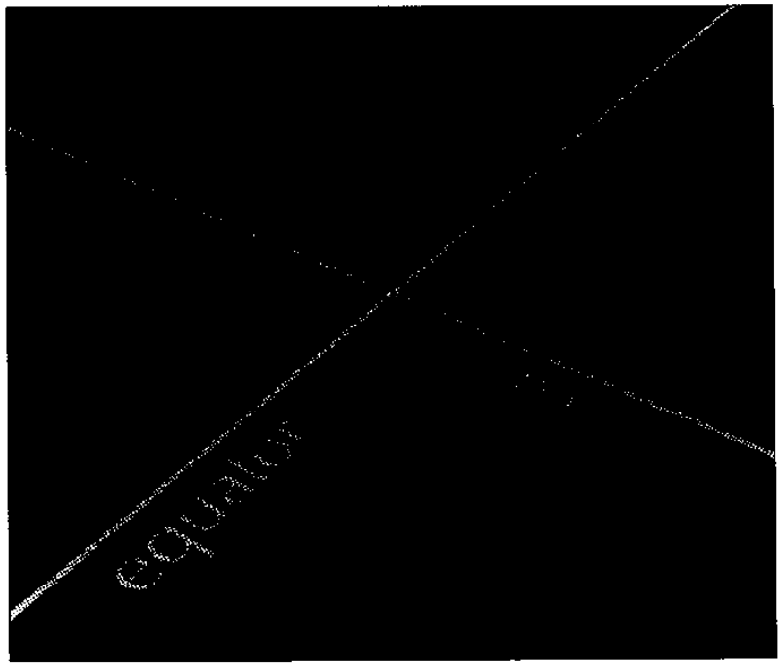


Figure 4. The initial configuration setup at $t = 0M$ for the distorted single black hole. ψ_4 is outlined with white contour lines, and ψ_0 is represented with a repetitive color texture map.

Our discussion begins with the dynamics of apparent horizons. We can visualize the 2D geometry of the apparent horizon's surface by removing it from the curved 3D space and embedding it into a flat 3D space in a way that preserves the horizon's geometry or curvature. This lets us see the surface more intuitively.

Figures 1 through 3 display the embedding diagrams and the time-line history of the apparent horizons (up to the time of the embedding) for the single distorted black hole and the colliding two black hole space-times. (These and all figures presented in this article were choreographed and rendered on a Silicon Graphics Iris using Wavefront Technology's Advanced Visualizer software package.) A color map helps visualize the curvature of these horizon surfaces. Blue represents regions of negative curvature (for example, the waist of a peanut has negative curvature), white is zero curvature (or a flat geometry), green is positive but less than 4π, red is exactly 4π (the normalized curvature of a perfect sphere), and transitions of pink to yellow denote curvatures greater than 4π. The shape and curvature of the current horizon is displayed at the bottom of each figure. At the top of each figure we present a time line with the time axis increasing horizontally to the right. The initial configuration is given at $t = 0M$, and the evolution runs to $t = 90M$, where M denotes the unit of time relative to the black hole mass. We constructed a history of events by piecing together polar slices of the embedding diagram at each time.

Figure 1 is the prolate single black hole horizon shown early in its history (when $t \sim 2M$). Figure 2 shows the same case at $t \sim 60M$. A comparison of the two figures clearly shows that the initial prolate configuration evolves into a spherical state. Notice that the horizon history at the top of Figure 2 vanishes at $t \sim 8M$. This is due to the extreme distortion suffered by the surface geometry at this time. Under these conditions the 2D curved surface cannot be embedded into flat space of one higher dimension.

Figure 3 shows a history of the head-on collision of two equal-mass black holes. The system starts off with two separated horizons surrounding each hole. As the holes move together, the horizons merge to form a single horizon surrounding the coalescing holes. The outer horizon then oscillates, but in a less violent manner compared to the highly distorted single black hole.

We have produced animations of both evolving apparent horizon geometries. The essence of these animations is evident from the stills in Figures 1



Figure 5. Same as Figure 4, except here we display the radiation pattern at $t \sim 20M$.

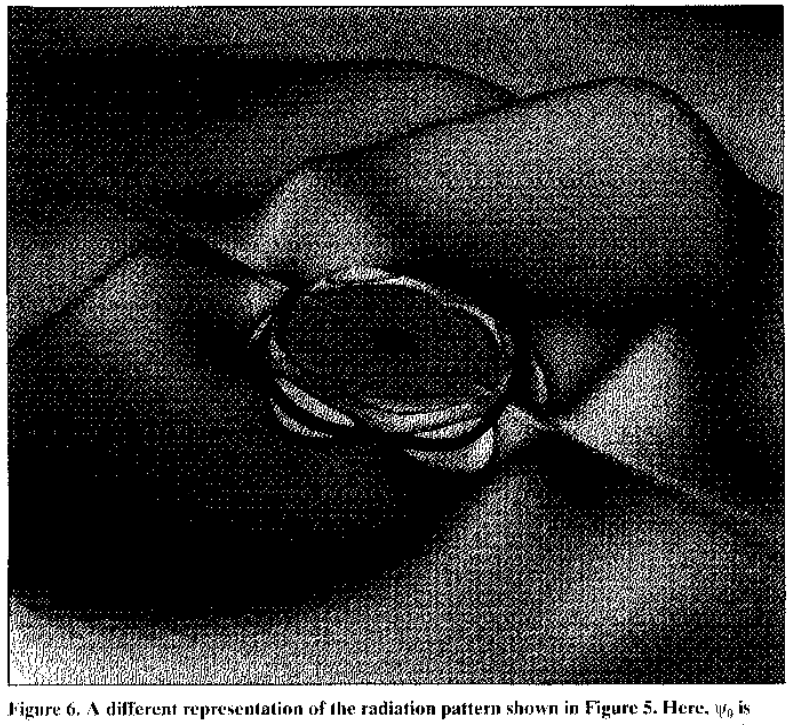


Figure 6. A different representation of the radiation pattern shown in Figure 5. Here, ψ_0 is still represented by the same color map as Figures 4 and 5, but ψ_4 is now mapped to a surface.

through 3. A number of generic features are common to these space-times:

1. The horizon oscillates at a frequency that is predominantly the normal mode frequency of the final spherical black hole.
2. The embedded surface begins from a highly distorted prolate/oblate configuration to become spherical.
3. The amplitude of oscillations damps away in time as the surface becomes more spherical.

These dynamical black hole space-times inevitably emit gravitational radiation as they decay to their spherical or stationary limit. To study this process, we can extract certain wave-like quantities from the local space-time curvature. In Figures 4 through 6 we display the Newman-Penrose scalars ψ_0 and ψ_4, which are suggestive of the inward and outward propagating gravitational waves. Although ψ_0 and ψ_4 are not strictly physically invariant, they provide insightful information about gravitational waves.

Figure 4 shows the initial data setup for the distorted single black hole at $t = 0M$. The labels refer to the axis of symmetry (analogous to the earth's north-south polar axis) and the equatorial plane. ψ_4 is outlined with equally spaced white contour lines, and ψ_0 is represented by a color map that varies smoothly from blue to gray to yellow in a repetitive fashion. The black circular line at the center marks the location of the apparent horizon.

As the system evolves, the black hole both absorbs and emits gravitational radiation. We display the radiation pattern at $t \sim 20M$ in Figure 5. Figure 6 presents another representation of this same radiation pattern. Here, ψ_0 is still displayed in color, but ψ_4 is now mapped to the surface height.

We are continuing to investigate new methods of data visualization. The images presented here are prototypical attempts to analyze important properties of the space-times and extract the relevant physics from large data sets. The experience gained in these 2D axisymmetric problems will be put to the test in fully 3D simulations of systems expected to occur with greater regularity in our universe. ❑

Further reading

Kip S. Thorne, "The Search for Black Holes," *Scientific American*, Vol. 231, No. 6, Dec. 1974, pp. 32-43.

Russell Ruthen, "Catching the Wave," *Scientific American*, Vol. 266, No. 3, March 1992, pp. 90-99.

Andrew Abrahams, David Bernstein, David Hobill, Edward Seidel, and Larry Smarr, "Numerically Generated Black Hole Space-times: Interaction with Gravitational Waves," *Physical Review D*, Vol. 45, May 1992, pp. 3,544-3,558.