Gravitational Radiation from Two Black Holes Colliding Head-On

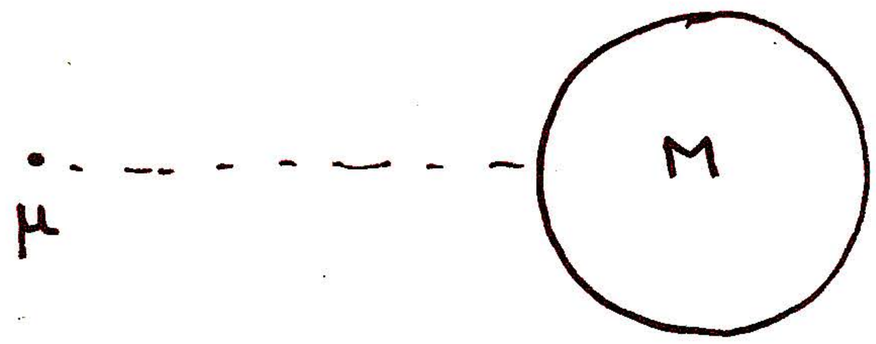
Larry Smarr, Princeton University and Harvard University

1975-1978

PDF created from Larry Smarr's original hand-colored transparencies by Charles Erwin

COLLIDING MASSES IN GENERAL RELAITITY

LARGE MASS RATIO



Linearize ∧ Einstein Equations About Nonlinear Dynamic

Background: Static Solution - Nonlinear

1 Ordinary Differential Equation
in one variable (r)

Davis, Ruffini, Press, Price (1971) - Head-on Schwarzschild
Detweiler (1978) - General Kerr

EQUAL MASS



Full Dynamic Einstein Equations - No Background

8 Coupled Partial Differential Equations
in 3 variables (R, z, T)

Eppley & Smarr (1978) - Head-on Schwarzschild

SUMMARY
THE TWO BLACK HOLE COLLISION

Why Problem is Important

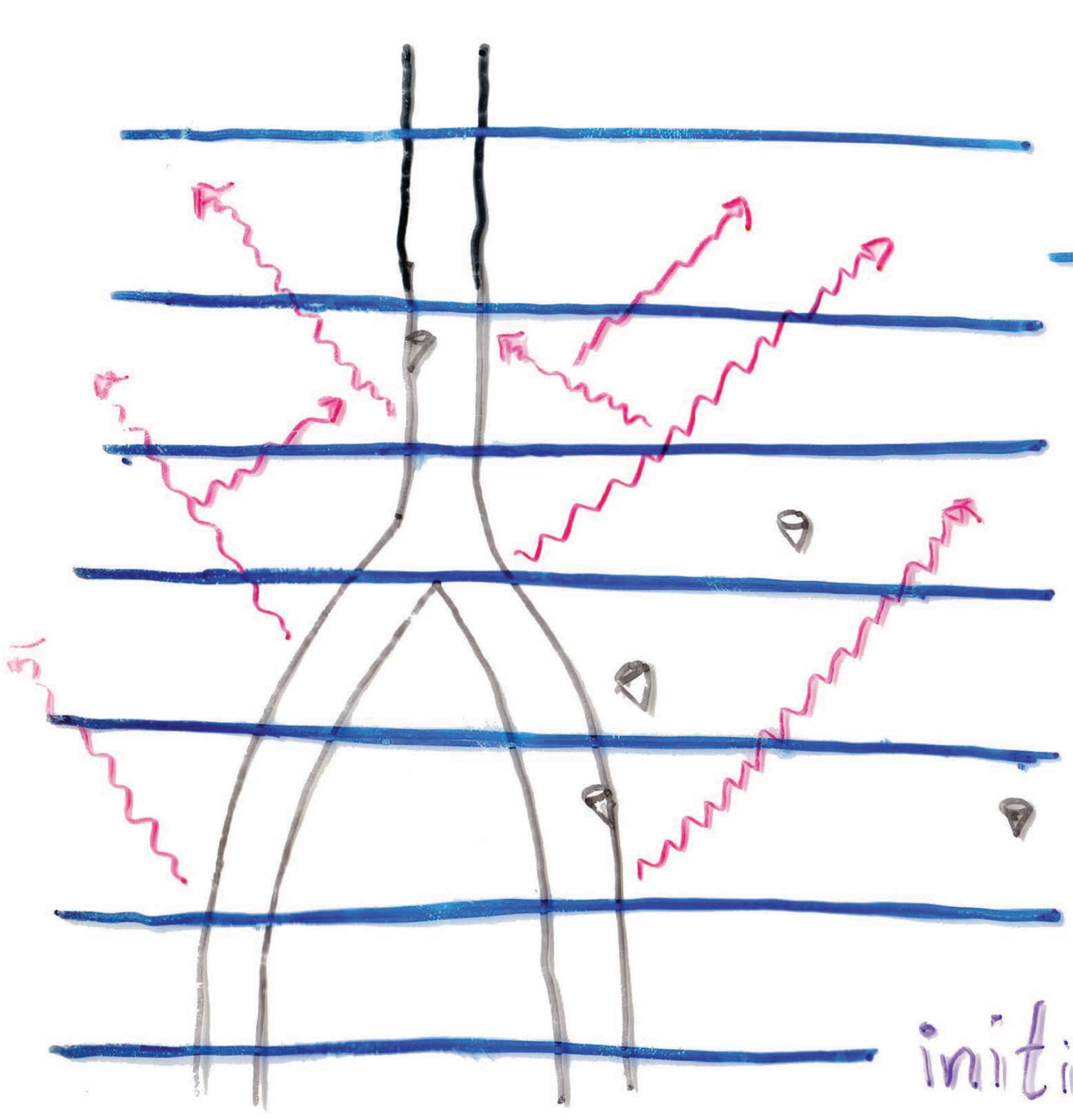
- Demonstrates Feasibility of Numerical General Relativity

- Contains Black Holes & Gravitational Radiation Interacting in Dynamic Strong Fields

- Provides Self Consistent Two Body Solution of Field Equations

- Explores Large Nonperturbative Deformations of Black Holes

Colliding Black Holes



L_0

initial separation

time

space

Final Hole

initial state

Holes at Rest

Major Physical Results

- Establishment View of Black Hole Collisions Upheld
- No Violation of Cosmic Censorship
- Gravitional Wave Generation Efficiency Calculated
- Upgrades Perturbation Results
- Any Solution of Einstein's Equations Can Be Found

Efficiency of Colliding Black Holes

Head-On Collision



$\frac{\Delta E}{Mc^2} = \varepsilon = .001$ (Smarr, Eppley)

$M = 2m \quad \mu = \frac{1}{4}m$

Perturbation Scaling

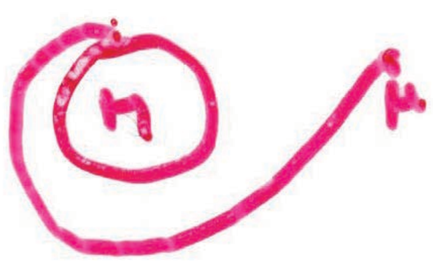


$\varepsilon = .01 \left(\frac{\mu}{M}\right)^2$ (Davis, Ruffini, Press, Price)

Scale up to equal masses

$\varepsilon \to .0006$ (Smarr)

Spiral Collision



$\varepsilon \to 2.0 \left(\frac{\mu}{M}\right)^2$ (Detweiler)

Scale up to equal masses

$\varepsilon \to .12$ (Smarr, Detweiler)



Full 3+1 calculation needs to be done

$\varepsilon \sim 10\% - 20\% \; Mc^2$

GRAVITATIONAL WAVE EMISSION

HYDRODYNAMIC GENERATION



$$L_{GW} \sim (\dddot{Q})^2 \sim \frac{(\Delta Q)^2}{(\Delta t)^6}$$ ← Sharp Deceleration Counts a Lot

- **Newtonian**
  - Thuan & Ostriker - Dust Spheroids
  - Novikov - Bounce Counts
  - Saenz & Shapiro - Pressure Spheroids
- **Post Newtonian**
  - Turner & Wagoner - Slow Rotation
- **Fully Relativistic**
  - Piran - Cylindrical Bounce
  - Smarr & Wilson - Axisymmetric Collapse

BLACK HOLE OSCILLATIONS

$\lambda \sim 19M$



- **Perturbed Black Holes**
  - Press - BH Vibrations
  - Davis, Ruffini, Press, Price - Ptl. Falls Into BH
  - Cunningham, Moncrief, Price - Star Collapse
- **Fully Relativistic**
  - Eppley, Smarr - BH Collisions

What Does Two Black Hole Collision Represent?

FIRST...

1) complete numerical calculation of a radiating, highly nonspherical, solution to Einstein's equations.

2) demonstrated feasibility of 2+1 numerical GMD.

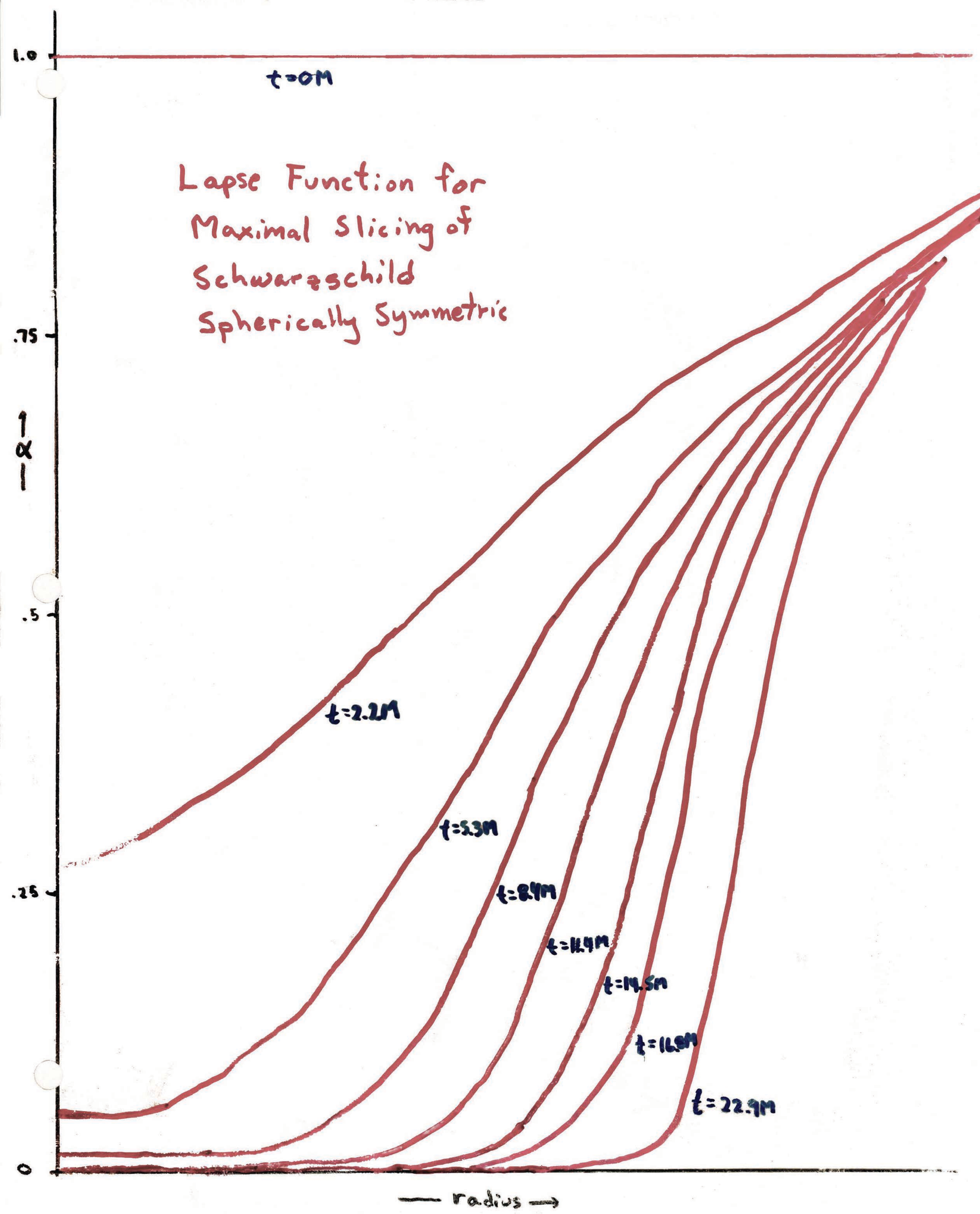
3) large deformation of black hole calculation.

4) self-consistent 2-body problem in G.R.

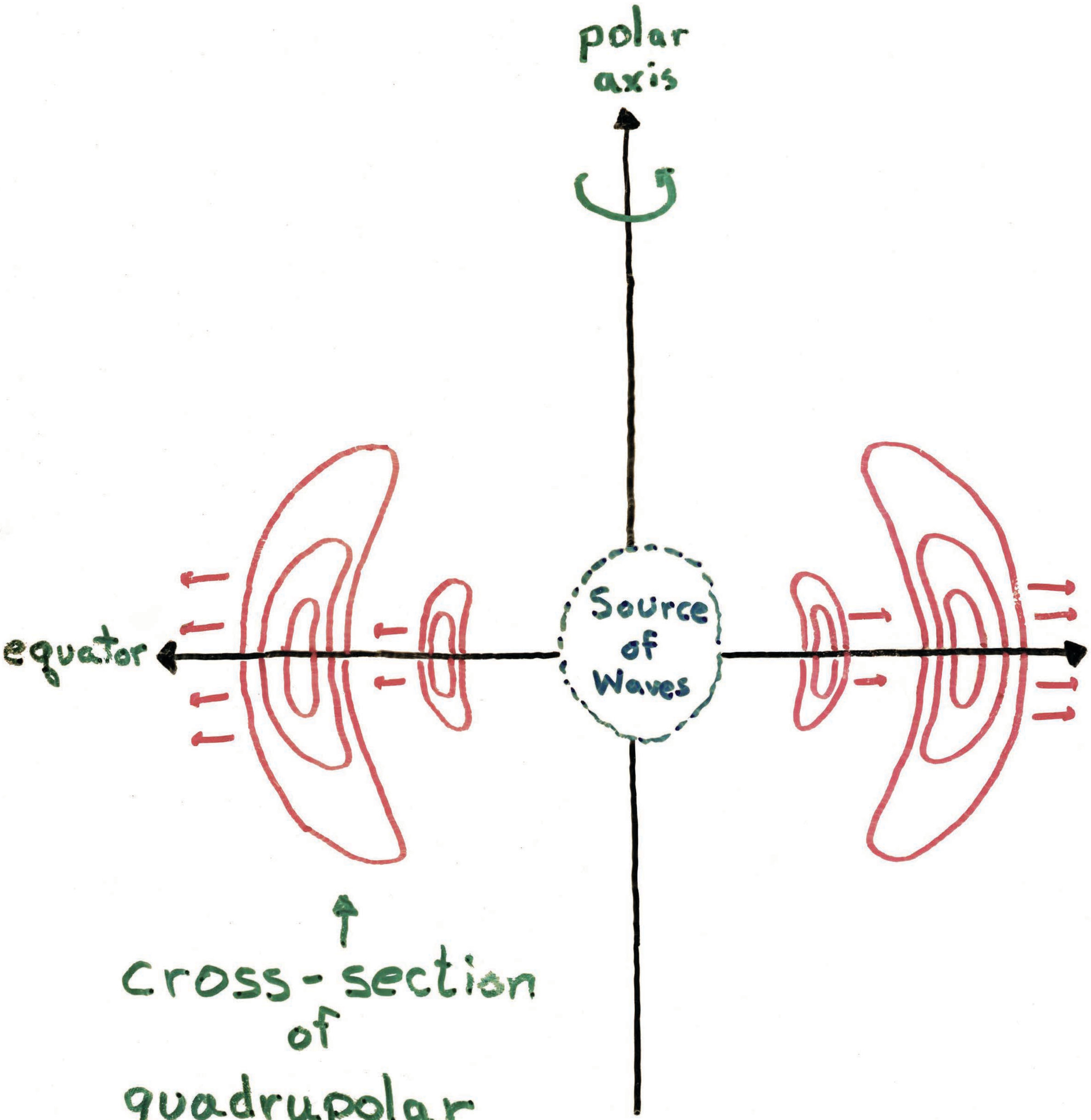
5) use of Ψ_4, B-R, B.S. radiation quantities in a nonperturbative spacetime

6) visualization of gravitational waves and strong field dynamics

Lapse Function for
Maximal Slicing of
Schwarzschild
Spherically Symmetric



Gravity Waves in Axisymmetric Spacetime



↑
cross-section of quadrupolar gravitational wave "donut"

RUN I
T = 34M

$\bar{P}_{BR}$
$\theta = \pi/2$

2.0
1.5
1.0
0.5
0.0
-0.5
-1.0
-1.5
-2.0

0.0 5.0 10.0 15.0 20.0 25.0 30.0 35.0 40.0

E -6

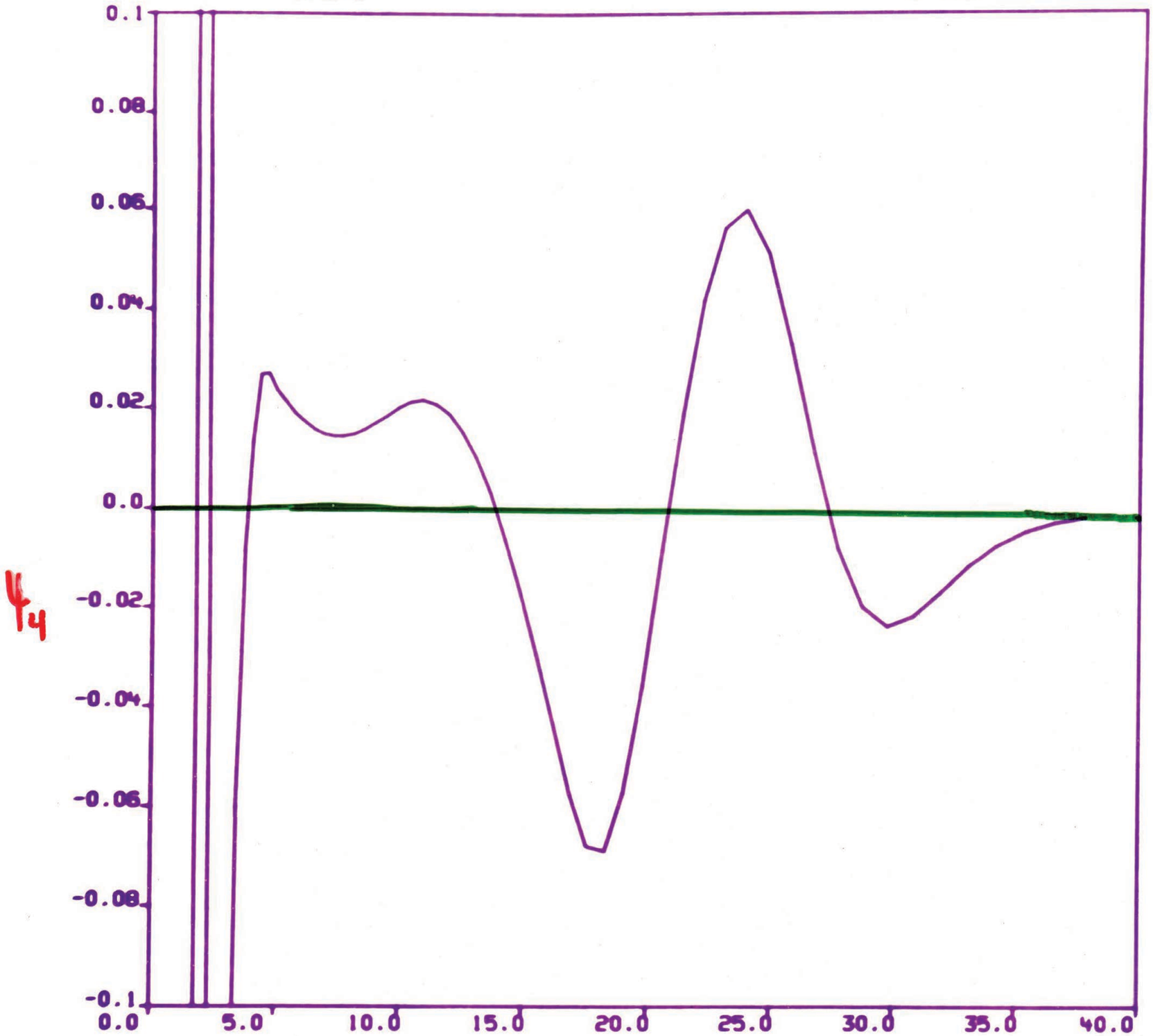
PETA T= 34.

((2011 P1(201 14. : MALL

DISPLAY 01/18/78U

RUN I
T = 34M

Note: error → $R^2 \times \Psi_4$



RUN I
T = 58M

$\vec{P}_{BR}$
$\theta = \pi/2$

2.0
1.5
1.0
0.5
0.0
-0.5
-1.0
-1.5
-2.0

reflected wave

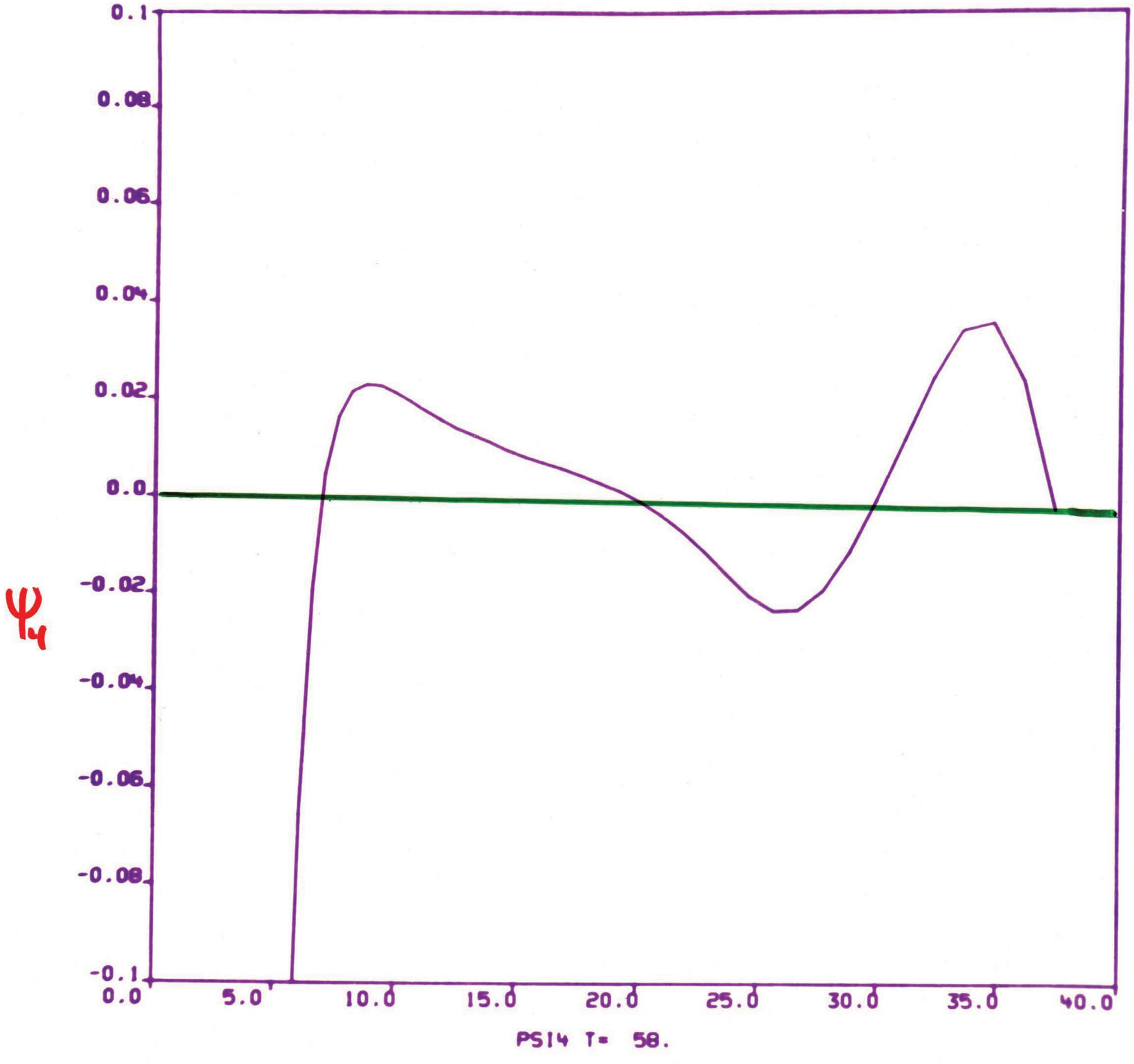
0.0 5.0 10.0 15.0 20.0 25.0 30.0 35.0 40.0

E -6

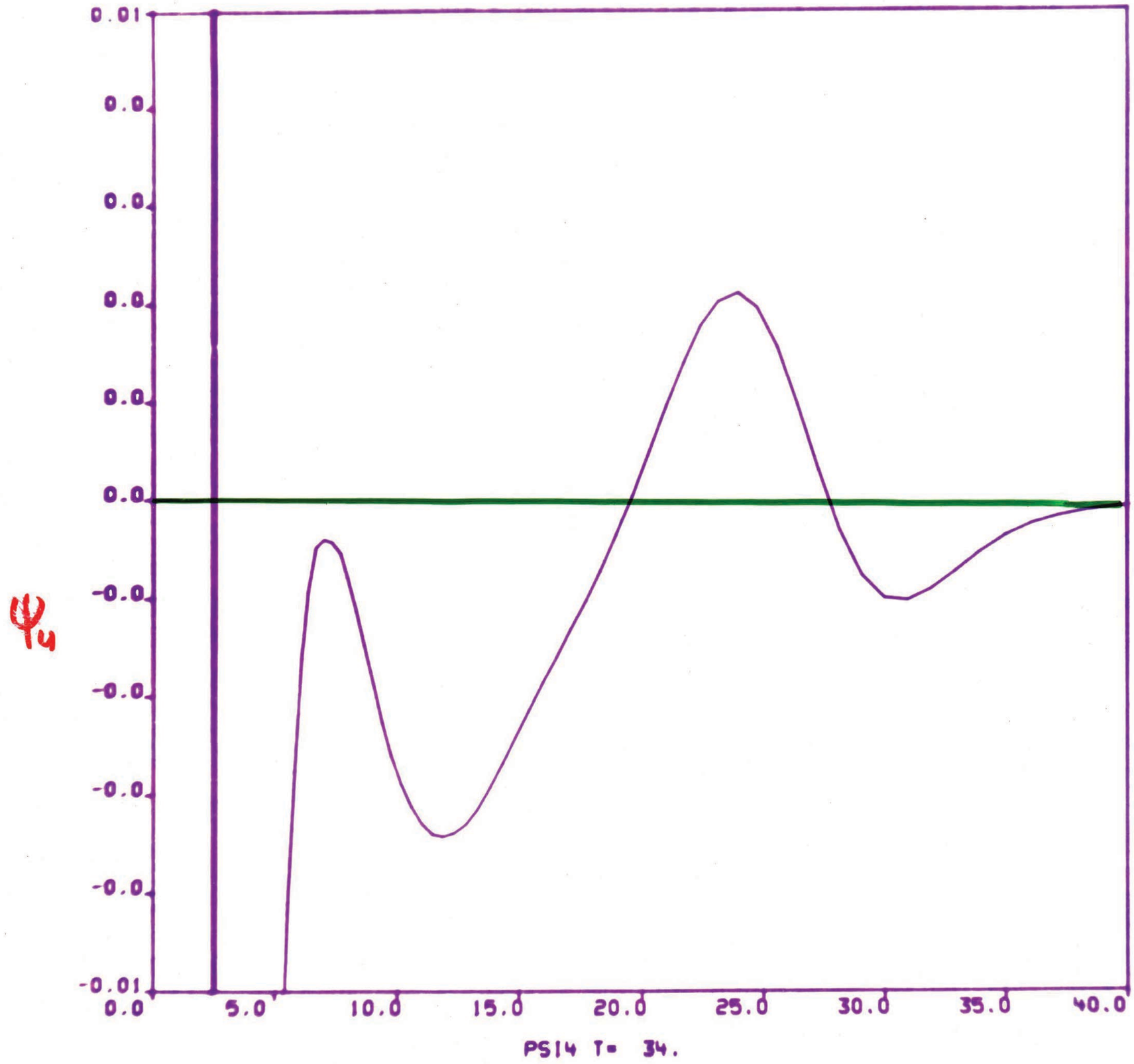
PETA T= 58.

RUN I
T = 58M

Note : error → $R^2 * \Psi_4$

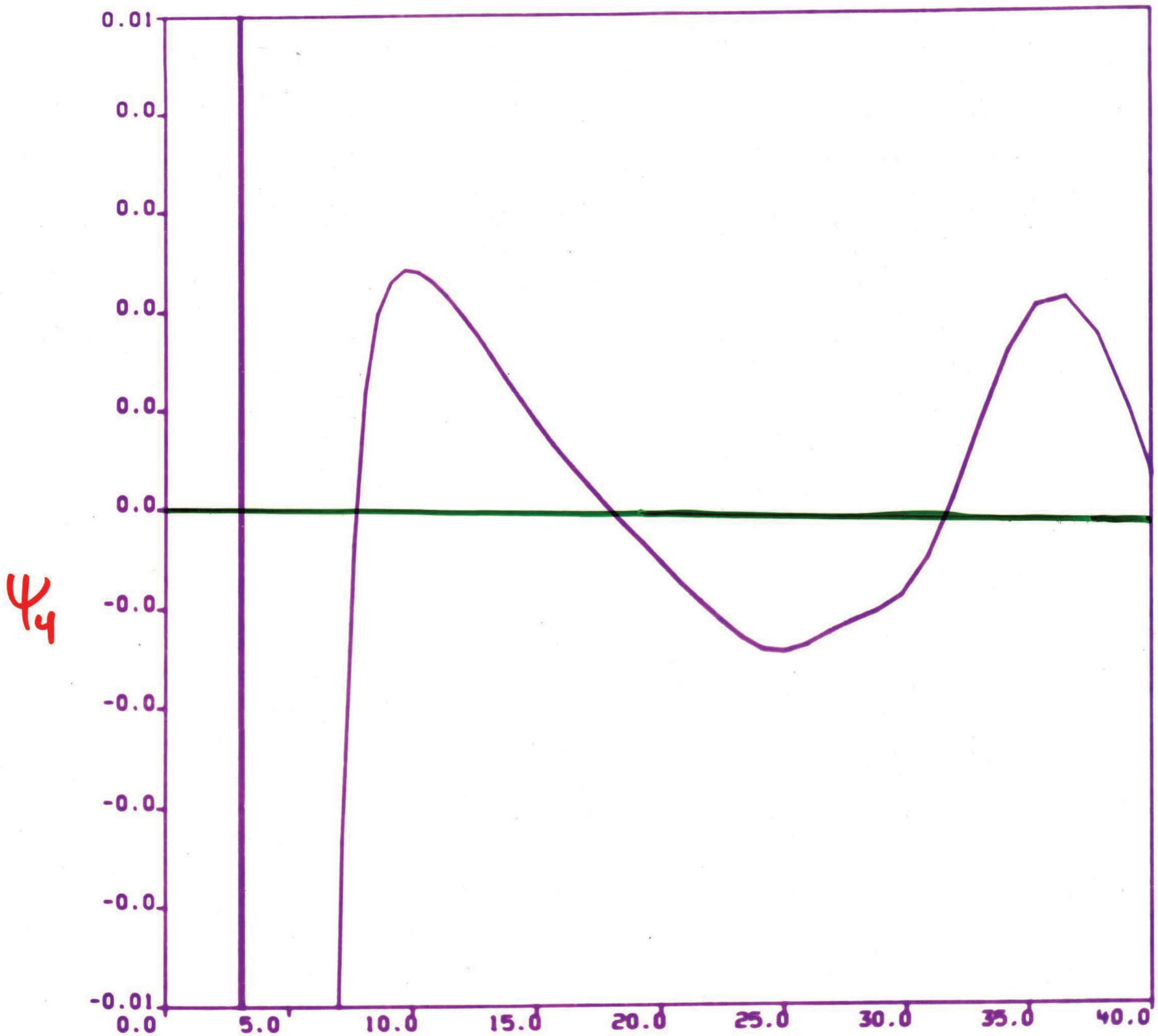


RUN II
T=34M


0.01
0.0
0.0
0.0
0.0
0.0
-0.0
-0.0
-0.0
-0.0
-0.0
-0.01
0.0
5.0
10.0
15.0
20.0
25.0
30.0
35.0
40.0
ψ_4
PS14 T= 34.

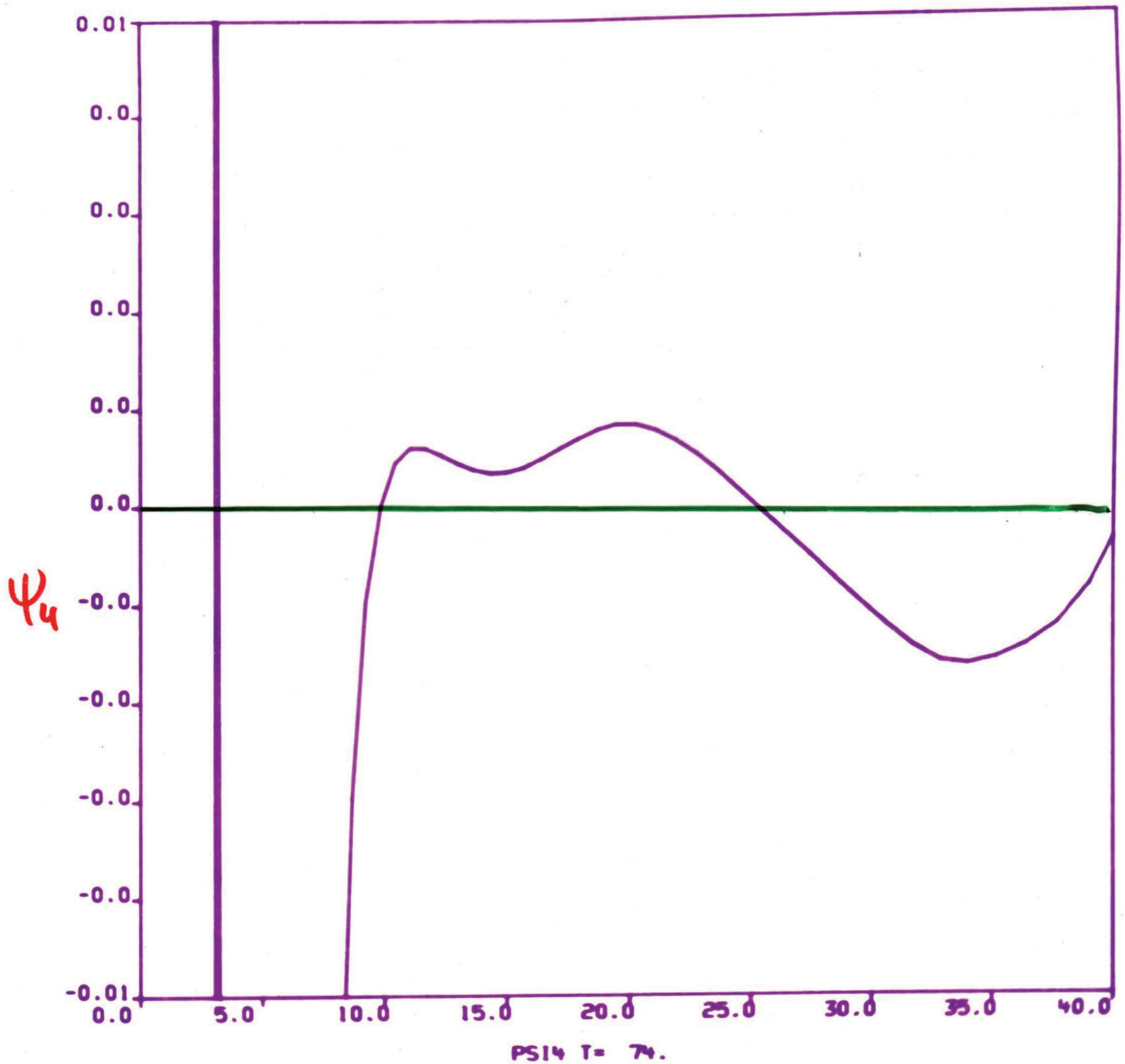
RUN II

T = 66M



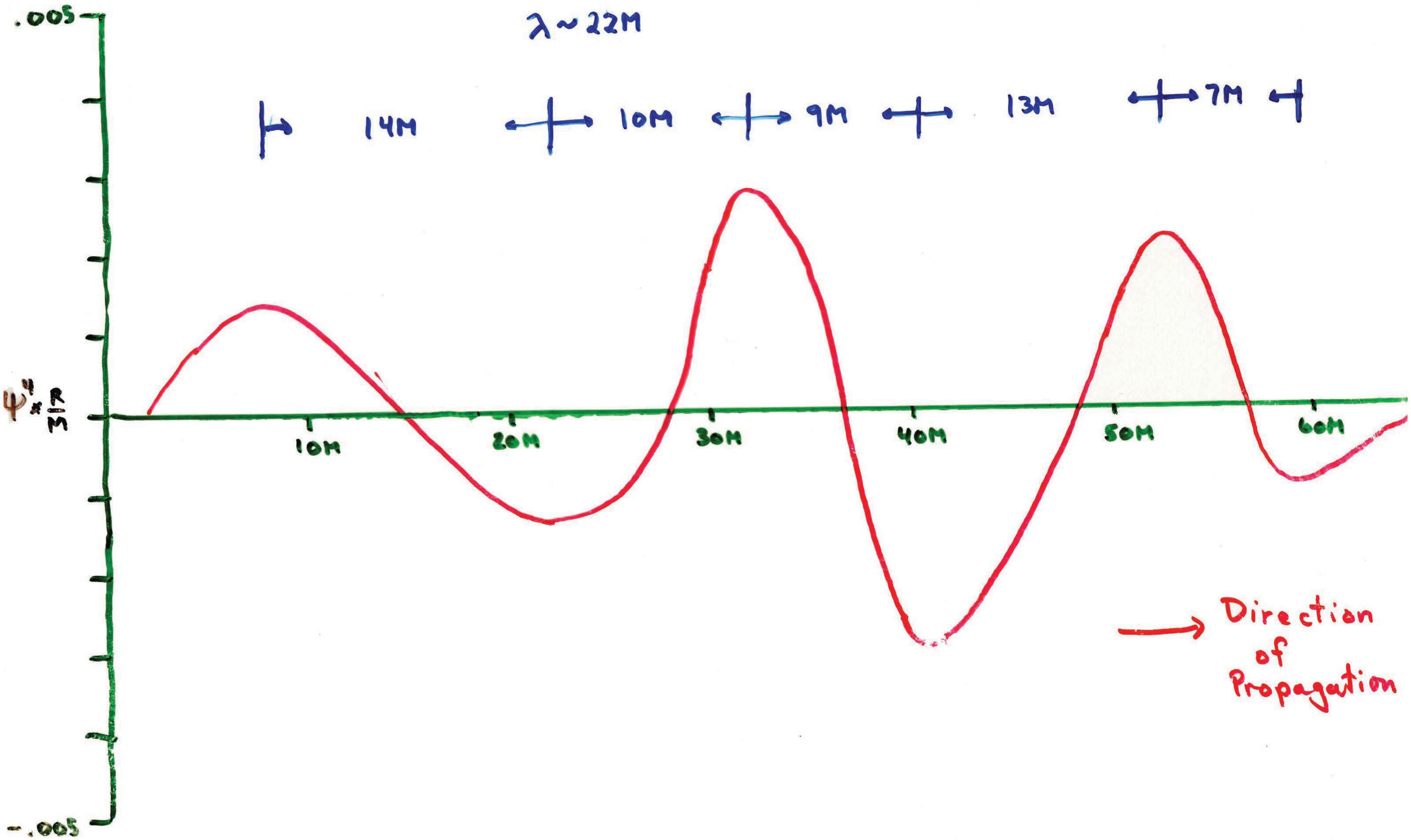
ψ_4

RUN II
T= 74M



PS14 T= 74.

RUN II Ψ_4


.005
Ψ_{4}^{22}
M1R
-.005
λ~22M
14M
10M
9M
13M
7M
10M
20M
30M
40M
50M
60M
Direction of Propagation

MAJOR RESULTS

1) Maximal slicing works in highly nonspherical strong fields. Lapse Collapse "generic".

2) Verifies Establishment picture of black hole collision.

3) Strong support to Cosmic Censorship.

4) Calculated grav. wave efficiency for head-on Collision of Schwarzschild black holes.

5) Event Horizon coalescence does <u>not</u> necessarily produce large amounts of grav. waves.

6) Verifies quasi-Newtonian approach and upgrading of perl. results.

7) Kip Conversion